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THIRD GRADE PERSPECTIVE

APPROVED BY



THE SCIENCE

AND ART

DEPARTMENT

Part. II.

BY H. J. DENNIS

17011 d. 45

THIRD GRADE PERSPECTIVE.

FIFTH



EDITION.

PART II.
SHADOWS AND REFLECTIONS.

SPECIALLY PREPARED

FOR THE USE OF ART STUDENTS

BY

H. J. DENNIS,

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1885.

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PREFACE TO PART II.

By a recent regulation of the Science and Art Department, I find **THIRD GRADE PERSPECTIVE**, if published as heretofore, will exceed the value of a **SECOND GRADE PRIZE**; therefore, in order that Art Students may not be disappointed, I have decided to issue this (**THE FIFTH**) and all future editions in two parts. The Department have consented to this arrangement, and have permitted the book to remain on their prize list, which, I am sure, will be much appreciated by those students who wish to continue their study of Perspective for the higher Examinations.

WEST KENSINGTON,
February, 1885.

Henry J. Dennis

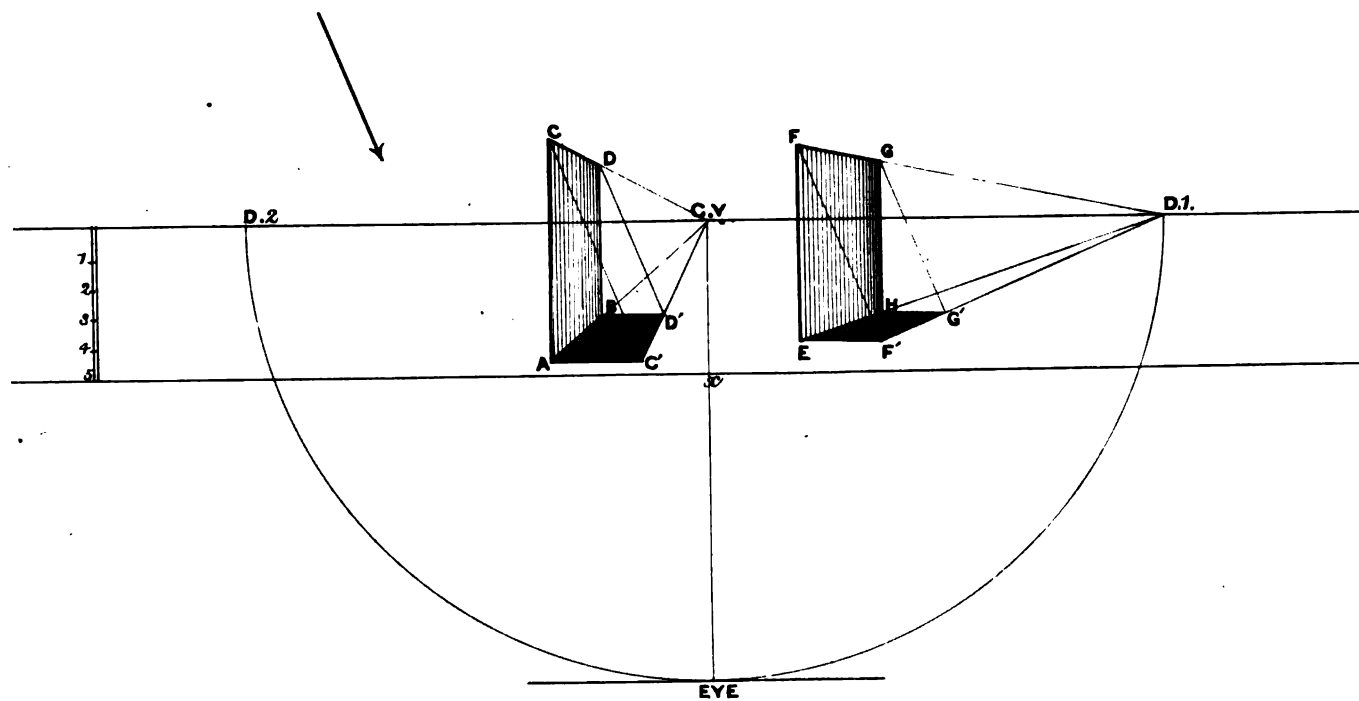


PLATE 32.

A B C D, E F G H, are the perspective representations of two squares, of 8' side, lying in vertical planes.

Required their shadows upon the ground-plane when the sun lies in the plane of the picture, its altitude being 60° .

Distance of eye in front of the picture, 15'.

Height of eye above the ground, 5'.

N.B. The Sun being in the plane of the picture, and at such a very great distance from the spectator, its position in that plane cannot be determined; and for the same reason, the rays of light, although really converging, appear as parallel lines.

When the Sun's position cannot be determined, the direction of the rays of light is always indicated by an arrow, thus ↘.

In order to obtain the shadow of the square, we must imagine *vertical planes parallel to the picture-plane* to contain the lines **A C**, **D B**, each plane also to contain a ray of light.

Now, suppose a vertical plane parallel to the picture-plane to contain the line **A C**, and the ray of light **C C'**; the intersection of this

vertical plane with the *plane of shadow* is the line **A C'**, the shadow of the side of square **A C**.

It should be observed that the side of square (**A C**); the intersection of the vertical plane with the plane of shadow (**A C'**); and the ray of light (**C C'**) form a right angled triangle; the base of which is the required shadow, and the hypotenuse is the ray of light.

The vertical plane which contains the line **D B** and the ray of light **D D'** cuts the plane of shadow in the line **B D'** which is the shadow.

Finally, join **C' D'** and it will be found to have **C V** for its vanishing point, because **C D** (the line which casts the shadow) is parallel to the plane of shadow (See General Rule, C.)

The shadow of the square **E F G H** is determined by the same method as previous one.

The vertical planes parallel to picture containing lines **E F**, **G H**, and the rays of light **F F'**, **G G'**, cut the plane of shadow in the lines **E F'**, **H G'**. Join **F' G'** to complete the shadow.

N.B. **F' G'** has **D 1** for its vanishing point because **F G** is parallel to plane of shadow and has same vanishing point. This rule will apply in every case.

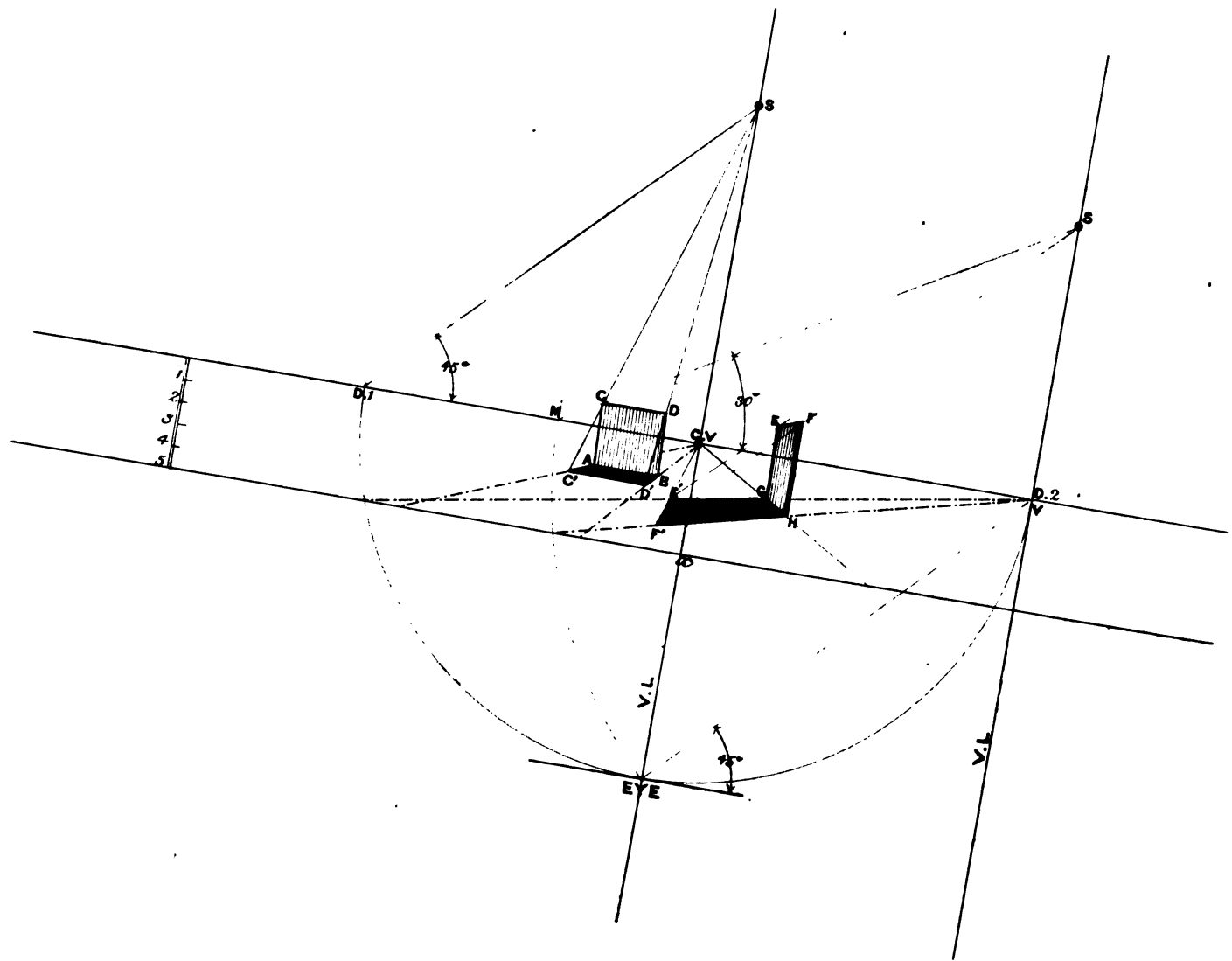


PLATE 33.

Distance of eye in front of the picture-plane, 15'.

Height of eye above the ground-plane, 5'.

A B C D, **E F G H**, are the perspective representations of two squares of 8' side lying in vertical planes.

I. Required the shadow of **A B C D** upon the ground when the Sun is *behind* the picture plane, lying in a vertical plane perpendicular to it, the altitude of the rays of light being 45° .

II. Required the shadow of **E F G H** upon the ground when the Sun lies *behind* the picture-plane in a vertical plane which makes 45° to right of spectator. Sun's altitude 30° .

N.B. When the sun is behind the picture plane, the rays of light ascend from the spectator; the front of the object is in shade and the cast shadow increases in breadth as it approaches the picture plane.

TO FIND THE SHADOW OF THE SQUARE **A B C D**.

Draw the **V L** of the vertical plane in which the Sun lies through **C V**, then make the Sun's altitude at **D 1**, or **D 2**, making 45° with the horizon, and produce it to meet the **V L** of vertical plane at **OS**.

Imagine vertical planes passing through **OS**, containing the lines **D B**, **A C**.

These vertical planes cut the plane of shadow in the chain-lines drawn through **A**, **B**, to **C V**; the rays of light, lying in the vertical planes meet the ground at points **C'**, **D'**.

Join **C'**, **D'**, which gives **A B C' D'**, the required shadow of the square.

TO FIND THE SHADOW OF THE SQUARE **E F G H**.

Draw the **V L** of the vertical plane which passes through the Sun's centre, through **D 2**. At **M** (the measuring point of **D 2**) make the angle of the Sun's altitude at 30° with the horizon.

The vertical planes passing through the Sun and containing the lines **F H**, **E G**, cut the plane of shadow (ground) in the chain-lines drawn from **G** and **H** to **D 2**; and the rays of light meet the ground on these chain-lines, at **E'**, **F'**.

Join **E' F'**, and the line will have **C V** for its vanishing point, because the line **E F** is parallel to the plane of shadow.

E' F' G H is the required shadow.

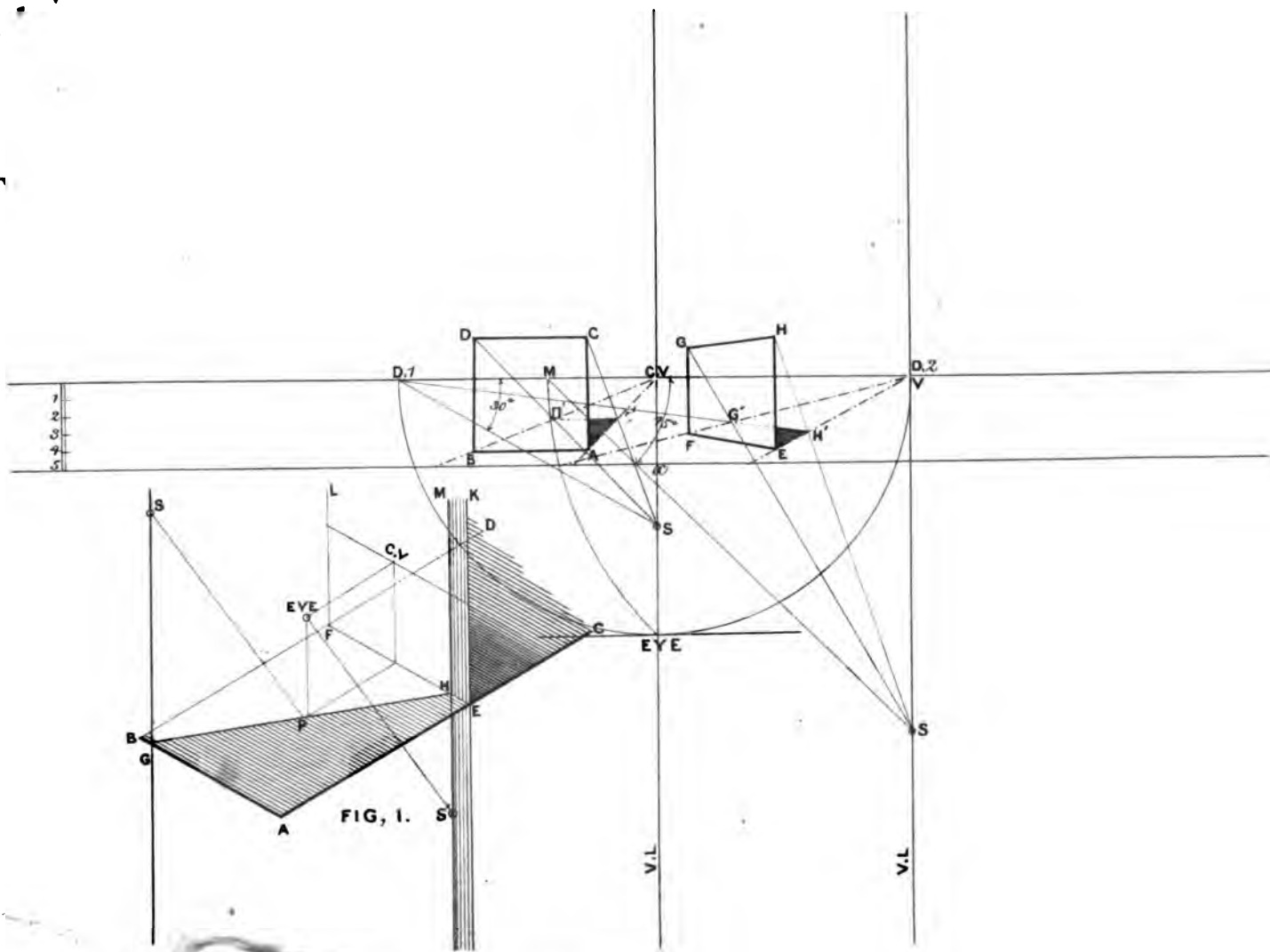


PLATE 34.

Distance of eye in front of picture-plane, 15'.

Height of eye above the ground-plane, 5'.

A B C D, **E F G H** are the perspective representations of two squares of 8' side lying in vertical planes.

I. Required the shadow of **A B C D** upon the ground-plane when the Sun is *in front* of the picture plane, in a vertical plane perpendicular to it. Altitude of Sun 30° .

II. Required the shadow of **E F G H** upon the ground-plane when the Sun is *in front* of the picture-plane, on left of spectator, and in a vertical plane which makes 45° with picture towards right. Sun's altitude 45° .

N.B. When the Sun is in front of the picture plane, it is far behind the spectator and cannot possibly be seen by him; the rays of light appear to descend from his eye, consequently their vanishing point is obtained below the horizon; and, if FIG. I. Plate 34 be referred to, the student will observe that if the Sun be really on left of spectator, its position on the picture plane must be found on the right.

Let **A B C D** (FIG. I.) represent the ground plane; **E F** picture-

line; **E F K L** the picture plane; **O** the spectator's eye, and **OS** the position of the Sun on the *left* of the spectator.

Imagine a vertical plane passing through the Sun's centre and the spectator's eye. This plane cuts the ground in the line **G H**, it also intersects the picture-plane in the line **H M**. *The line H M becomes the vanishing line of the plane which contains the Sun.*

Now, suppose **OS P** to be a ray of light lying in the vertical plane above referred to. Draw from **O** (spectator's eye) the vanishing parallel of the ray of light, it will be found to meet the picture, on the vanishing line **H M** produced below the ground plane, at **OS'**.

Point **OS'** is evidently the vanishing point of the Sun's rays, and proves what has been said in the above NOTE.

TO FIND THE SHADOW OF **A B C D**.

The Sun being in a vertical plane perpendicular to the picture-plane, has its **V L** passing through **C V**; and, because the Sun is in front of the picture-plane, its altitude must be drawn at **D 1**, or **D 2**, below the horizon, at 30° with it, meeting **V L** of plane of sun at **OS**.

Imagine vertical planes passing through the Sun and containing the sides of square **A C**, **B D**. The intersections of these planes with the

ground are shown by the chain-lines drawn through **A** and **B** to **C V**.

Now draw the rays of light from **C** and **D** to the Sun ($\odot S$) intersecting the ground on the chain-lines, at **C'** and **D'**. Join **C'**, **D'**.

A B C' D' is the shadow of the square, only a small portion of which is seen by the spectator.

TO FIND THE SHADOW OF **E F G H**.

The vanishing line of the plane containing the sun should be drawn through **D 2**.

Find **M** (the measuring point of **D 2**).

At **M**, draw the vanishing parallel of the Sun's altitude making

45° below the horizon, and produce it to meet **V L**, at $\odot S$, the position of the Sun on the picture plane.

As in preceding example, imagine vertical planes passing through the Sun containing the sides of square **E H**, **F G**. These vertical planes intersect the ground in the chain-lines drawn through **E**, **F** to **D 2**; the rays of light meet the ground at **G'** and **H'** and determine the length of required shadow.

The line **G H** and its shadow **G' H'** have the same vanishing point; because the plane of shadow and the line are parallel. (See General Rule C.)

That portion of the shadow which is visible to the spectator is tinted.

PLATE. 3

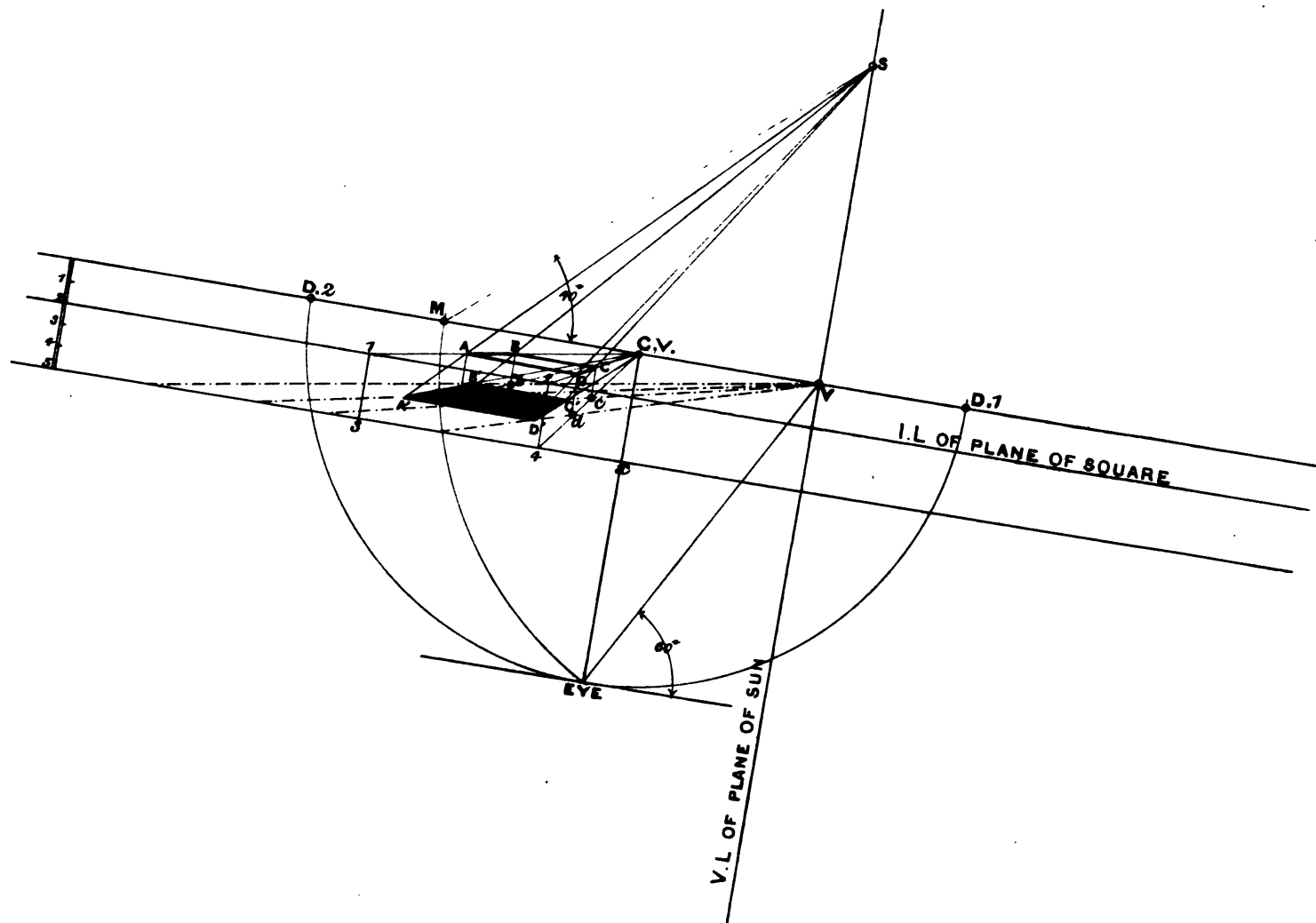


PLATE 35.

Distance of eye in front of picture-plane, 15'.

Height of eye above the ground-plane, 5'.

A B C D is the perspective representation of a square of 8' lying in a horizontal plane 2' below the level of the spectator's eye.

Required its shadow upon the ground when the Sun lies *behind* the picture, in a vertical plane which recedes at an angle of 60° towards right. Sun's altitude 40° .

It is necessary to find four points immediately below the corners of the square upon the ground-plane.

Trace out **B A, C D**, by **C V** to meet **I L** of plane of square at points 1 and 2. From 1 and 2 let fall perpendiculars to meet the ground-line at points 3 and 4. Join 3 and 4 to **C V**. These lines lie upon the ground immediately below **A B, C D**, therefore if we let fall perpendiculars from **A, B, C, D**, to meet them, we shall have four

points (**a, b, c, d**) upon the ground under the corners of the square.

Now, find the vanishing line of the vertical plane containing the Sun at 60° with the picture towards right.

V is the vanishing point of the intersections with the ground of the vertical planes containing the rays of light. Find **M**, the measuring point of this vanishing point.

At **M** draw the vanishing parallel of the Sun's altitude making an angle of 40° with the horizon, produce it to meet **V L** of plane of Sun at **OS**.

Imagine vertical planes to pass through the Sun and contain the corners of the given square, their intersections with the ground-plane are shown by the chain-lines drawn through **a, b, c, d**, to **V**.

Draw the rays of light from **OS** through **A, B, C, D**, to meet the ground on the chain-lines at **A', B', C', D'**, which is the representation of the shadow required.

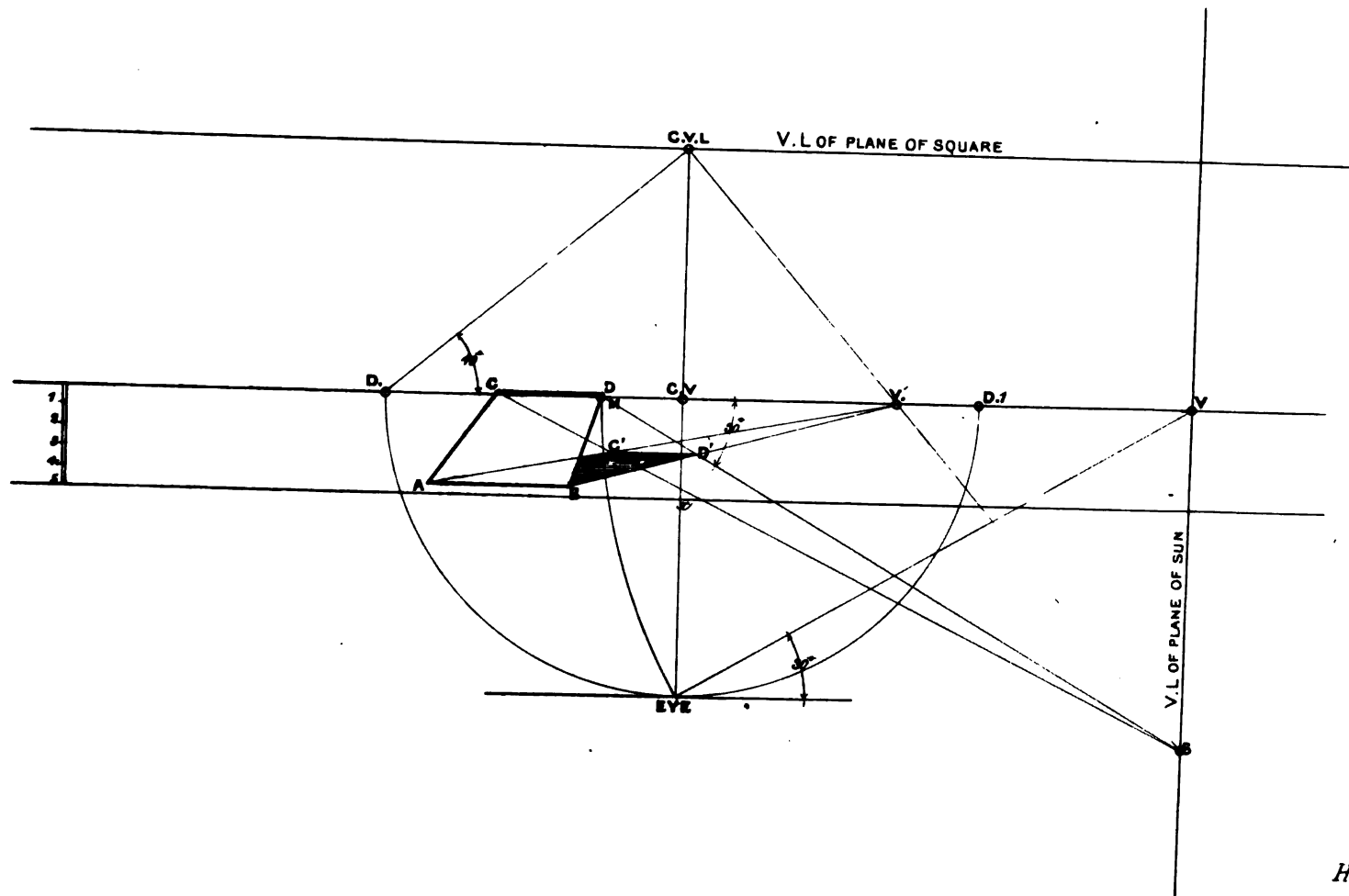


PLATE 36.

Distance of eye in front of the picture-plane, 15'.

Height of eye above the ground-plane, 5'.

A B C D is the perspective representation of a square of 8', lying in a plane inclined directly upwards from the spectator at an angle of 40° with the ground-plane.

Required its shadow upon the ground plane when the Sun is in front of the picture plane, to the left of the spectator, and lying in a vertical plane which makes 30° with the picture-plane. Sun's altitude 30° .

Find the **V L** of plane of Sun at 30° to *right*. Point **V** is the vanishing point of the intersection of the plane of Sun with the ground.

At **M** draw the vanishing parallel of the Sun's altitude, *below* the horizon at an angle of 30° with it, giving $\odot S$ on **V L** of plane of Sun.

N.B. In order to find the shadow of the square we must imagine planes to pass through the Sun and contain the lines whose shadows

are required. The intersections of these planes with the *plane of shadow* will give the shadows.

Imagine a plane to pass through the Sun containing the line **B D**. The **V L** of this plane is obviously a line drawn through **C V L**, and $\odot S$, because these points are respectively the vanishing points of the line, and Sun. The intersection of this plane with the *plane of shadow* (ground-plane) vanishes to **V'**, because the vanishing lines of the planes intersect at this point; therefore join **B** to **V'**.

Now, if we imagine a ray of light drawn from **D** to $\odot S$, and lying in the plane containing the line and Sun, its intersection with the *plane of shadow* must evidently be on **B V'**, at **D'**.

The shadow of the line **A C** is found upon the line **A V'**, because it represents the intersection with plane of shadow, of a plane passing through $\odot S$ containing **A C**.

The ray of light from **C** to $\odot S$ gives the shadow **C'**.

Join **C' D'**.

It should be observed that the planes passing through the Sun containing the lines **A C**, **B D**, are parallel, hence their intersections with plane of shadow have same vanishing point, **V'**.

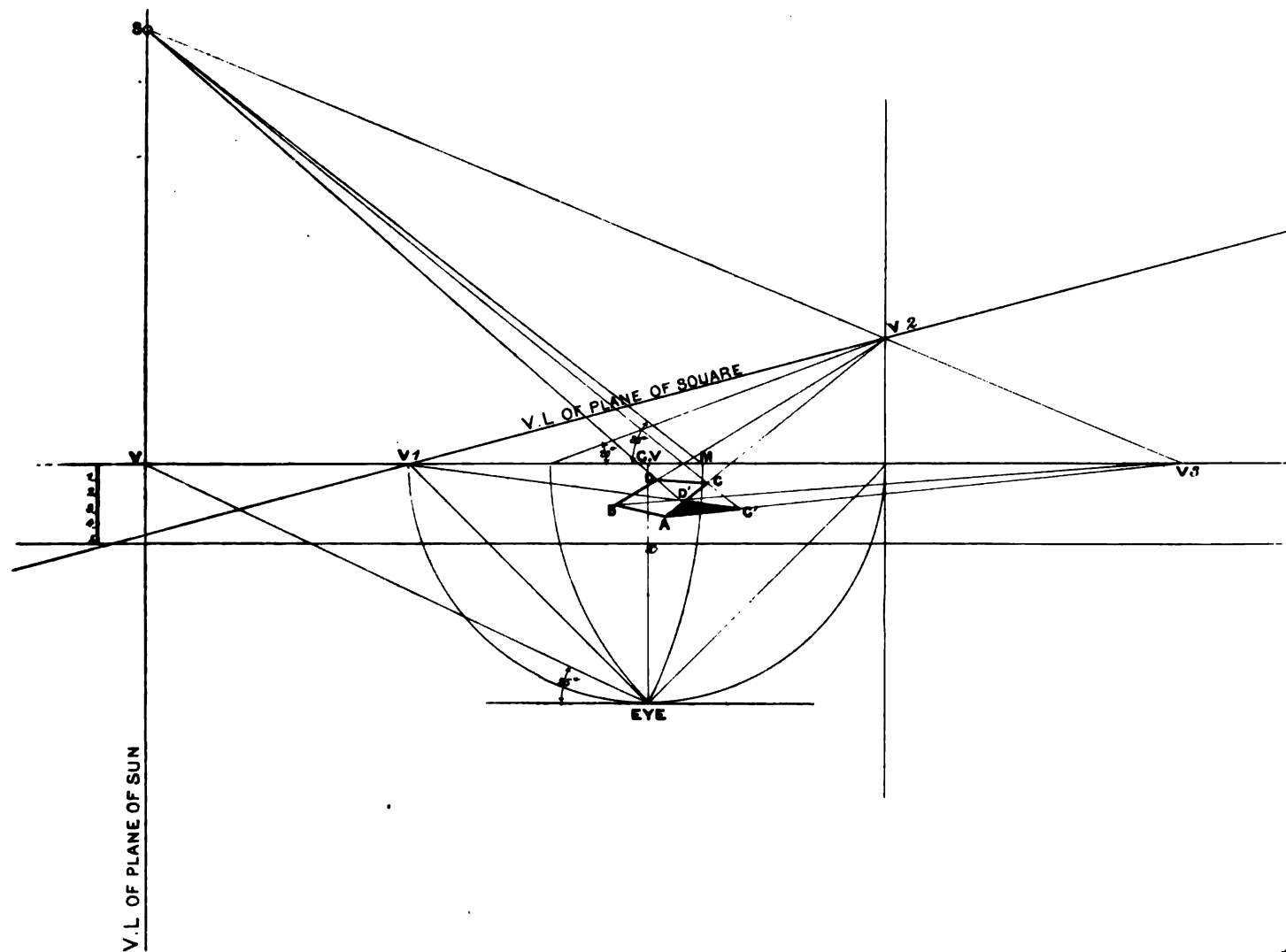


PLATE 37.

Distance of eye in front of the picture-plane, **15'**.

Height of eye above the ground-plane, **5'**.

A B C D is the perspective representation of a square of **8'**, lying in an oblique ascending plane, its direction with the picture-plane is **45°** to the left of spectator, and inclination to ground **20°**.

Required its shadow upon the ground-plane, when the Sun is behind the picture-plane, in a vertical plane which vanishes at **25°**, to left, with the picture plane. The Sun's rays make an angle of **38°** with the ground.

Find the **V L** of plane in which the Sun lies. Point **V** is the vanishing point of its intersection with the ground.

At **M** make the Sun's altitude at **38°** with the horizon, giving **○S**, the vanishing point of the Sun.

Join **V 2** (the vanishing point of the lines **A C, B D**) to **○S**, and produce the line to meet the **V L** of the ground (plane of shadow) at **V. 3**. See General Rule **B** Plate **32**.

*The line **○S, V 3**, really represents the **V L** of two parallel planes passing through the Sun containing the sides of square **A C, B D**.*

V 3 is the vanishing point of the shadows of the lines **A C, B D**; because it is the vanishing point of the intersections with the ground, of the planes containing the rays of light and sides of square; therefore, draw from **A** and **B** to **V 3**, also draw the rays of light from **○S** through **D** and **C** to meet the ground at **C', D'**.

Join **C', D'**, and the shadow **C' D'** will be found to have the same vanishing point as **C D** (See General Rule **C**, Plate **32**.)

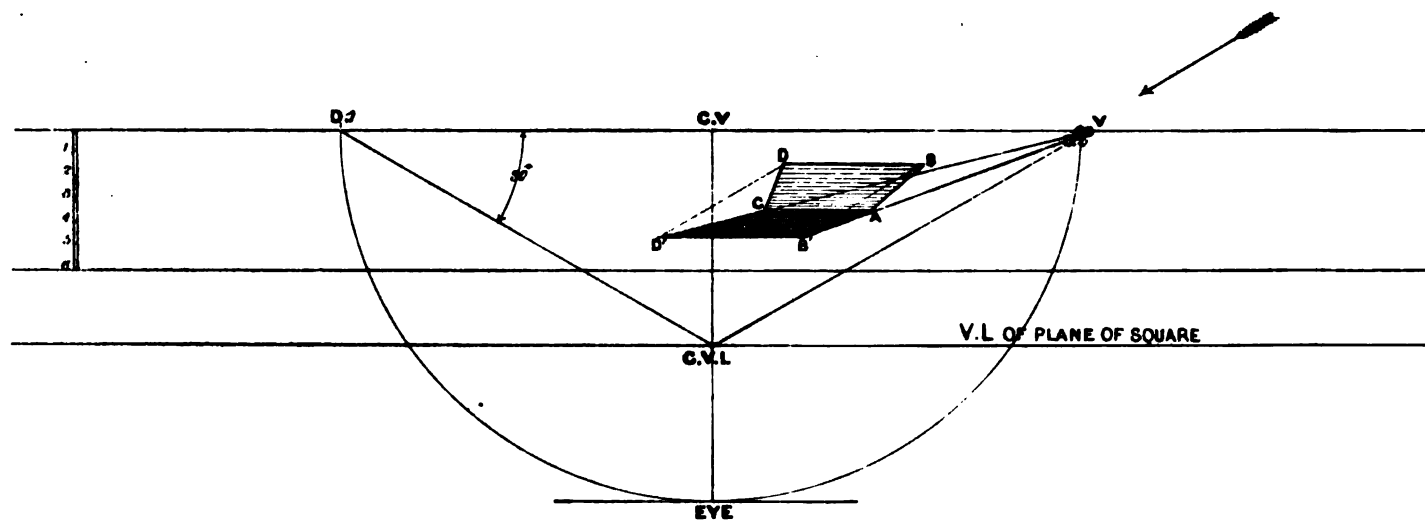


PLATE 38.

Distance of eye in front of the picture-plane, 16'.

Height of eye above the ground-plane, 6'.

A B C D is the perspective representation of a square of 8' lying in a plane which *descends directly* from the spectator at an angle of 30° with the ground-plane.

Required the shadow of the square upon the ground-plane when the Sun is in the plane of the picture, on the right of the spectator, at an altitude of 30° .

In this case the Sun has no vanishing point, but the arrow indicates the direction of the rays of light.

In order to find the shadows of the lines **A B, C D**, it is necessary to imagine planes passing through them and the Sun; the intersections of these planes with the *plane of shadow* determine the shadows.

The **V L** of the plane which passes through the Sun and the line

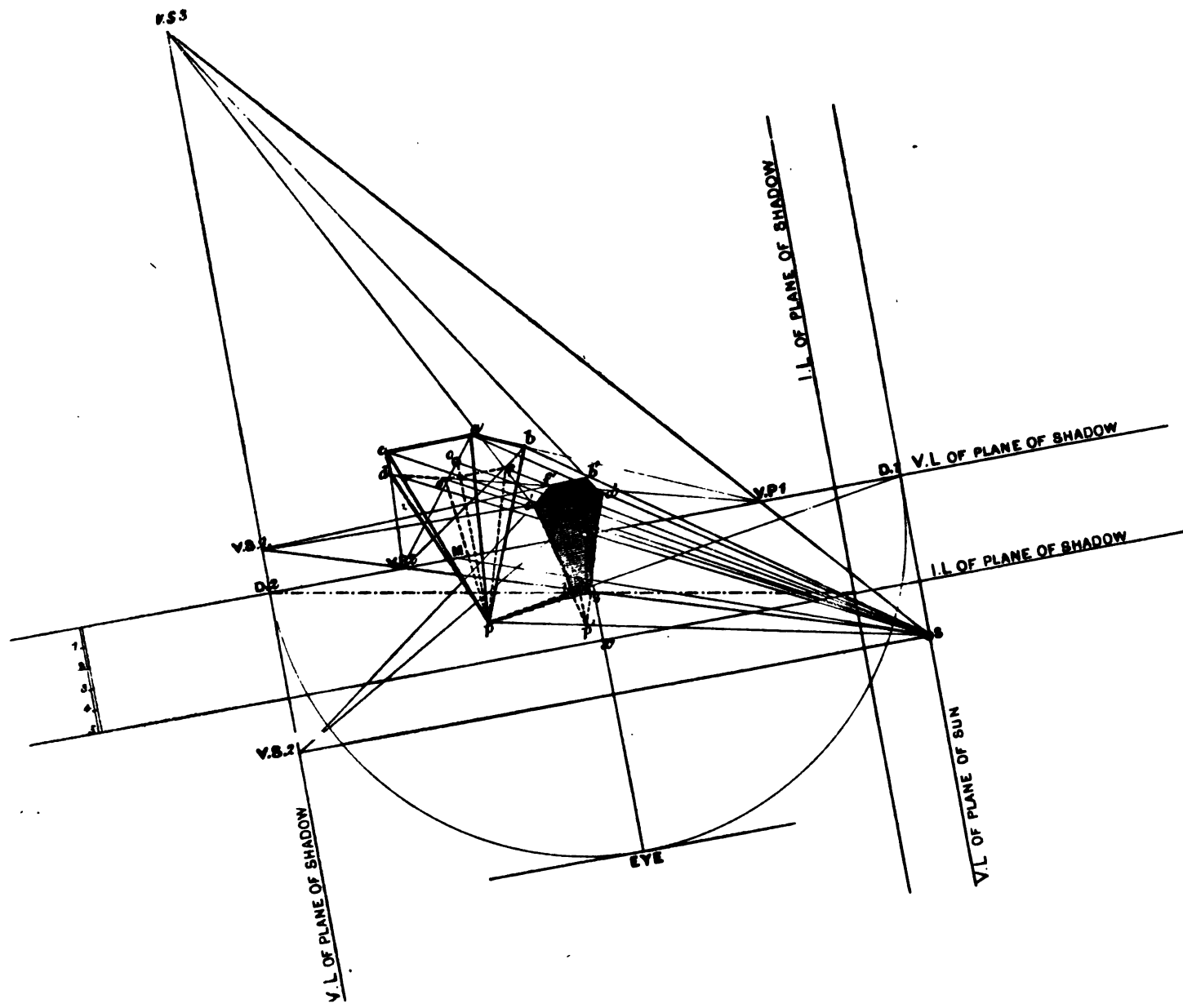
A B is found by drawing a line from **C V L** parallel to the rays of light. It will be observed that this **V L** cuts the **V L** of *plane of shadow* (ground-plane) at **V**, therefore, **V** must be the vanishing point of the intersection of the two planes, and consequently the vanishing point of the shadow of **A B**.

Join **A** to **V** and produce the line forward; then, if the ray of light **B B'** be drawn from **B** parallel to the given ray, the shadow **A B'** will be determined.

Now imagine a second plane to pass through the Sun, parallel to first plane, and contain the line **C D**.

Since the first and second planes are parallel, their intersections with the *plane of shadow* must be also parallel; therefore, join **C** to **V** and produce the line forward. If **DD'** be drawn parallel to the given ray of light the shadow **C D'** will be determined.

Join **D' B'**, which completes the required shadow.



H.J. Dennis

PLATE 39.

Distance of eye in front of the picture-plane, 16'.

Height of eye above the ground-plane, 5'.

I. A hexagonal pyramid stands upon the ground-plane on its apex at **S** on left of spectator and **S** beyond the picture-plane. Two edges of the base are parallel to the picture. Height of pyramid 12'. Edge of base 5'.

II. Required the shadow of this pyramid upon two planes, viz: the ground-plane, and a vertical plane which makes 45° with the picture-plane towards left, its **IL** being at 12' on spectator's right. The Sun is to be in front of the picture-plane, on left of spectator, in a vertical plane which makes 45° with the picture. Sun's altitude 20° .

Through **D. 1** draw the **V. L** of the plane containing the Sun, and at **M** draw the vanishing parallel of the rays of light at 20° with the horizon, *below it*, which gives **⊙ S** the position of the Sun.

The shadow of the axis of the pyramid should be now obtained.

Imagine a vertical plane to pass through the Sun and the axis, the intersection of this plane with the *two shadow planes* determines the shadow.

The **V L** of the imaginary vertical plane above referred to, cuts the **V L** of first shadow plane (ground-plane) at **D. 1**; which is the

vanishing point of the intersection of the two planes, consequently the vanishing point of the shadow of the axis upon the ground-plane.

Join **p** to **D. 1**, this line meets the intersection of the *two shadow planes* at point **3**, which is the termination of the shadow of the axis upon the ground-plane, its continuation upon the *vertical plane of shadow* is found by drawing a vertical line from **3**, and the ray of light **o, ⊙ S** to meet it at **O**.

P, 3, O is the shadow of the axis upon the two shadow planes.

We now proceed to find the shadow of the diagonal (**a e**) of the base of pyramid upon the *vertical plane of shadow*.

By referring to the General Rule it will be seen that the vanishing point of **a e** (**V P. 2**) must be joined to **⊙ S** and the line produced to meet **V L** of vertical plane of shade at **V S. 1**. **V S. 1**, is the vanishing point of the shadow of the line **a e**; therefore, join **O** to **V S. 1**, produce the line to meet the rays of light **a ⊙ S, e ⊙ S**, at **a'** and **e'**.

N.B. The line **V S. 1, ⊙ S**, really represents the vanishing line of a plane passing through the Sun and the line **a e**; its intersection with the *vertical plane of shadow* is shown by the line **V S. 1, e' a'**, and the student should observe that the shadow falls upon this intersection.

The shadows of the edges of the base of the pyramid should now be determined upon the same principle as that of the diagonal **a e**.

Through **V P. 1** (the vanishing point of **a b**) draw a line to **⊙ S** and produce it to meet **V L** of *vertical plane of shadow* at **V S. 3**.

V S. 3 is the vanishing point of the shadow of the line **a b**.

For, if we imagine a plane to pass through the Sun and **a b**, it would obviously cut the vertical plane of shadow in the line **V S. 3 a'**.

Determine **b'** by drawing the ray of light **b ⊙ S** to meet the line **V S. 3 a'**.

The edge of base, **b f**, is parallel to the diagonal **a e**; therefore, its shadow will vanish to **V S. 1**. Join **b'** to **V S. 1**, the ray of light **f ⊙ S** will determine the length of shadow **b' f**.

N.B. The edges **a c**, **f e**, are parallel to the picture-plane and have no vanishing point; therefore, if we imagine a plane to pass through the Sun and the lines, the **V L** of such planes will be found by drawing a line through **⊙ S** parallel to **a c**, **e f**, to meet **V L** of vertical plane of shadow at **V S. 2** which is the vanishing point of the shadows **e' f'**, **a' c'**, and their lengths are determined by drawing the rays of light **e ⊙ S**, **c ⊙ S**.

Then, because the edge **c d** is parallel to the diagonal **a e**, its

shadow must have the same vanishing point; therefore, join **c'** to **V S. 1**, and draw the ray of light **d ⊙ S** which gives the length of the shadow **d' e'**.

The edge **d e** is parallel to **a b**, therefore, join **d'** to **V S. 3** and if the student has worked very accurately he will find this line gives **e'** where it was previously obtained.

TO FIND THE SHADOWS OF THE LONG EDGES.

Imagine the *vertical shadow plane* produced below the ground, and find the position of the shadow of the apex of pyramid upon it.

Produce the line **O' 3** downwards, then draw the ray of light **p ⊙ S** which meets the shadow of axis at **p'**.

Join **p', e'**; **p', a'**; these lines meet the intersection of the planes of shadow at points **1, 2**, the *termination of the shadow of pyramid upon each shadow plane*.

To complete the shadow upon the ground, join **1, 2**, to the apex (**p**) of the pyramid.

PLATE 40.

Distance of eye in front of the picture-plane, 17'.

Height of eye above the ground-plane, 6'.

a b c d e f g h is the perspective representation of an octagon of 5' edge, lying in a plane perpendicular to the picture, inclined to ground-plane at an angle of 30°.

Required the shadow of the octagon upon two planes; viz., the ground-plane and a vertical plane perpendicular to the picture-plane, the **I L** of which is at 10' on right of spectator. The sun lies in the plane of the picture, on the left of the eye, and has an altitude of 45°.

The intersection of the *two shadow planes* is shown by the chain-line **C V O**.

The intersections of the two shadow planes with the *plane of the octagon* are represented by the chain-lines **C V O'**, **C V O''**.

The arrow indicates the direction of the rays of light.

TO FIND THE VANISHING POINTS OF THE SHADOWS OF THE EDGES OF
THE OCTAGON.

Imagine a series of planes passing through the sun and containing the edges of the octagon, the intersections of these planes with the shadow planes will determine the shadow of the octagon.

A plane passing through the sun and the side of the octagon, **c b**, is parallel to the picture, and its intersections with the *two shadow planes* and the plane of the octagon are shown by the chain-lines **2, 3**; **3, 1**; **1, 2**.

Now, imagine the rays of light **c c'**, **b b'**, to lie in the plane which passes through the sun. These rays meet the shadow planes at points **c'** and **b'**, and determine the length of the shadow.

It should be observed that the line **c b** casts a shadow upon both shadow planes; and, since these planes are at right angles the shadow must be of same form.

C' 3 is that portion of the shadow which falls upon the ground-plane, while **3 b'** represents the remainder upon the vertical shadow plane.

N.B. The student should bear in mind that the sun in its present position has no vanishing point.

The **V L** of a plane passing through the sun and the line **a b** is found by drawing a line through **V P 1** (vanishing point of **a b**) *parallel to rays of light, because the sun has no vanishing point.*

The **V L**, if produced both ways, will meet the **V L** of the shadow-planes at points **V S 1**, *the vanishing points of the shadows.*

Join **b'** to the point **V S 1**, on **V L** of vertical shadow plane, the

shadow of $a b$ must lie somewhere upon this line, because it represents the intersection of the plane passing through sun and $a b$, with the shadow-plane. Draw the ray of light $a a'$ which gives the length of the shadow $a' b'$.

A plane passing through the sun and line $a h$ (edge of octagon) intersects the vertical shadow-plane in a line drawn from a' to $C V$, because the line and shadow-plane are parallel to each other, both being *perpendicular* to the picture-plane. The ray of light $h h$ gives the length of shadow $a' h'$.

Now, imagine a plane passing through the sun and $g h$, the $V L$ of such plane is obtained by drawing a line through $V P 2$ parallel to the rays of light, which if produced to meet the $V L$ of each shadow-plane will give $V S 2$.

The line $g h$ only casts its shadow upon the vertical shadow-plane, therefore, join h' to $V S 2$, *on $V L$ of this shadow-plane*, and determine the length of the shadow by drawing the ray of light $g g'$.

The edges of the octagon $f g, c d$ are parallel, therefore their shadows must be parallel.

Draw $g' 4$ parallel to $b' 3$, and $4 f$ parallel to $3 c'$. The ray of light $f f$ gives $f' 4 g'$ as the shadow of $f g$ upon the two shadow-planes.

Imagine a plane to pass through the sun and line $f e$, the $V L$ of this plane is drawn through $V P 1$ (vanishing point of $f e$) parallel to rays of light, meeting the $V L$ of ground at $V S 1$, *the vanishing point of the shadow of $e f$ upon the ground-plane*.

Join f' to $V S 1$, and produce the line towards the picture-plane, the length of the shadow will be obtained by drawing the ray of light $e e'$.

Because $d e$ is parallel to the shadow-plane (ground-plane) its shadow will vanish to the vanishing point of $d e$. Join e' to $C V$ and the ray of light $d d'$ gives the length of the shadow.

Finally, imagine a plane to pass through the sun and line $d c$; its $V L$ should be drawn through $V P 2$ parallel to rays of light to meet the $V L$ of ground at $V S 2$; join d' to $V S 2$, and if the line be produced towards the picture it will meet at C' ; and prove the accuracy of the work.

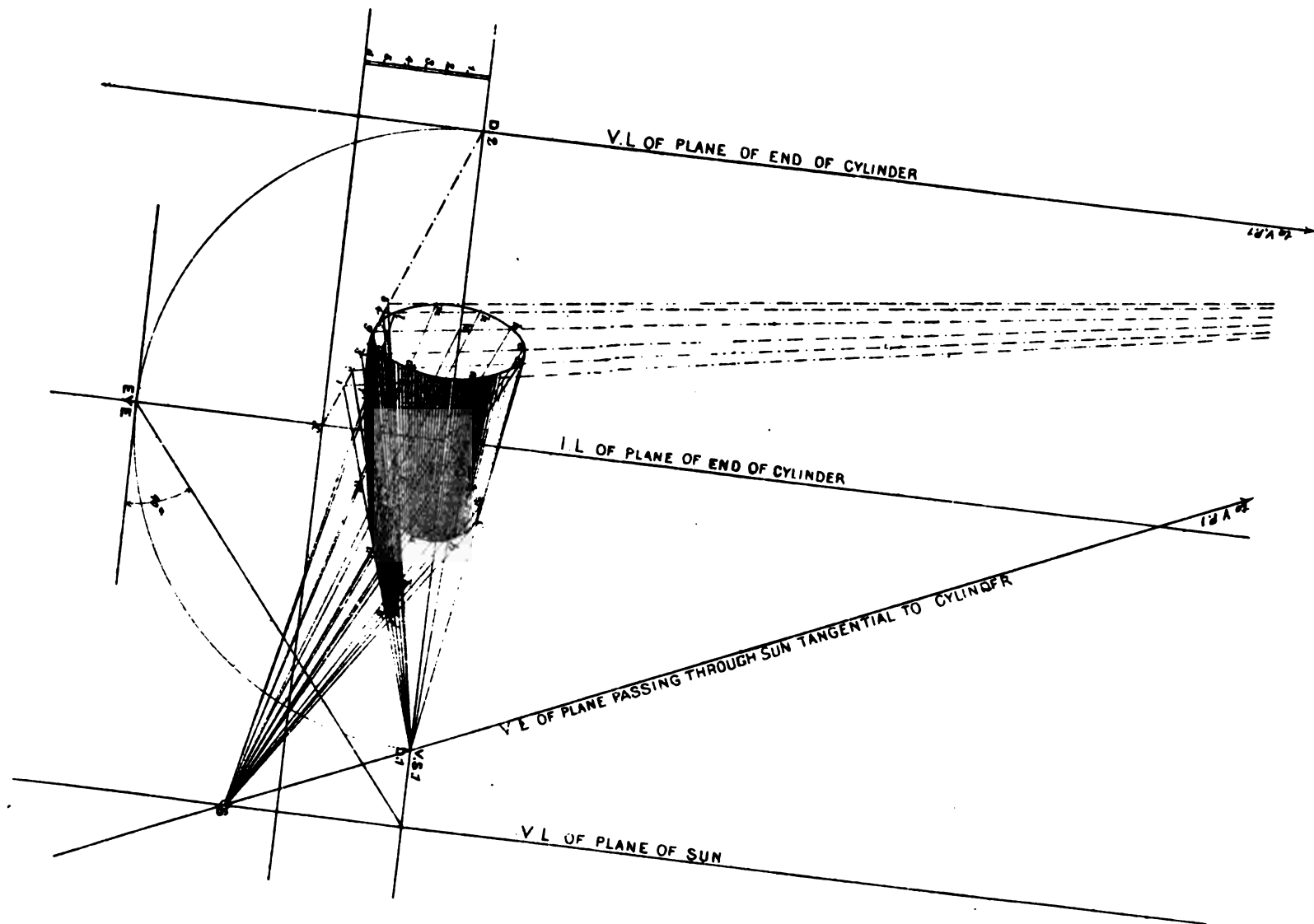


PLATE 41.

Distance of eye in front of the picture-plane, 15'.

Height of eye above the ground-plane, 6'.

A right cylinder 20' long, 10' 6" diameter lies on its side upon the ground-plane, the planes of its ends recede from the picture-plane at 45° towards spectator's left. The I L of near end of cylinder coincides with the *principal visual ray*, and the point nearest the picture-plane in the circumference of this end is 5' 6" from the I L of the plane in which it lies.

Required the shadow of the cylinder upon the ground-plane when the sun is in front of the picture-plane, on the left of the spectator, in a vertical plane which makes 40° with the picture-plane.

Sun's altitude 20°.

Obtain the V L of an imaginary plane passing through the sun tangential to the nearest side of cylinder, also the intersections of this plane with the ground, and the plane of end of cylinder.

If we join the vanishing point of the sun (⊙ S) to the vanishing point of the length of cylinder (D 1), we shall have the required vanishing line.

The tangential plane intersects the *ground* in the line 1, V S 1, and the plane of the end of cylinder in the chain-line which passes from 1 through a to V P 1. Its line of contact with the curved surface of the

cylinder is the line a b, which is the commencement and darkest part of the shadow. It is the line which separates the light from shadow.

Since the tangential plane meets the plane of shadow (ground) upon the line 1, V S 1, it is evident the shadow of a b must lie somewhere upon this line: draw rays of light from a, b to ⊙ S meeting the ground at a' b'.

A series of planes parallel to the tangential plane passing through the sun should be drawn.

The intersection of these planes with the plane of near end of cylinder are shown by the chain-lines 2 d c, 3 f e, g h, 4 l k, and 5 m. Their intersections with the shadow-plane (ground) are shown by the lines drawn from points 2, 3, g, 4, 5, to V S 1.

Now, if we imagine rays of light drawn from the points c d, e f, g h, k l, and m, to ⊙ S, to cut the plane of shadow on the intersections of the planes in which they respectively lie, we shall have points in the cast shadow of the cylinder.

For example, points c and d lie in the plane which cuts the ground in the line 2, V S 1; to meet this line we merely draw rays of light from c, d, *at both ends of cylinder*, which determine four points in the cast shadows of the ends of the cylinder.

N.B. That part of the shadow which cannot be seen by the spectator is shown by dotted lines.

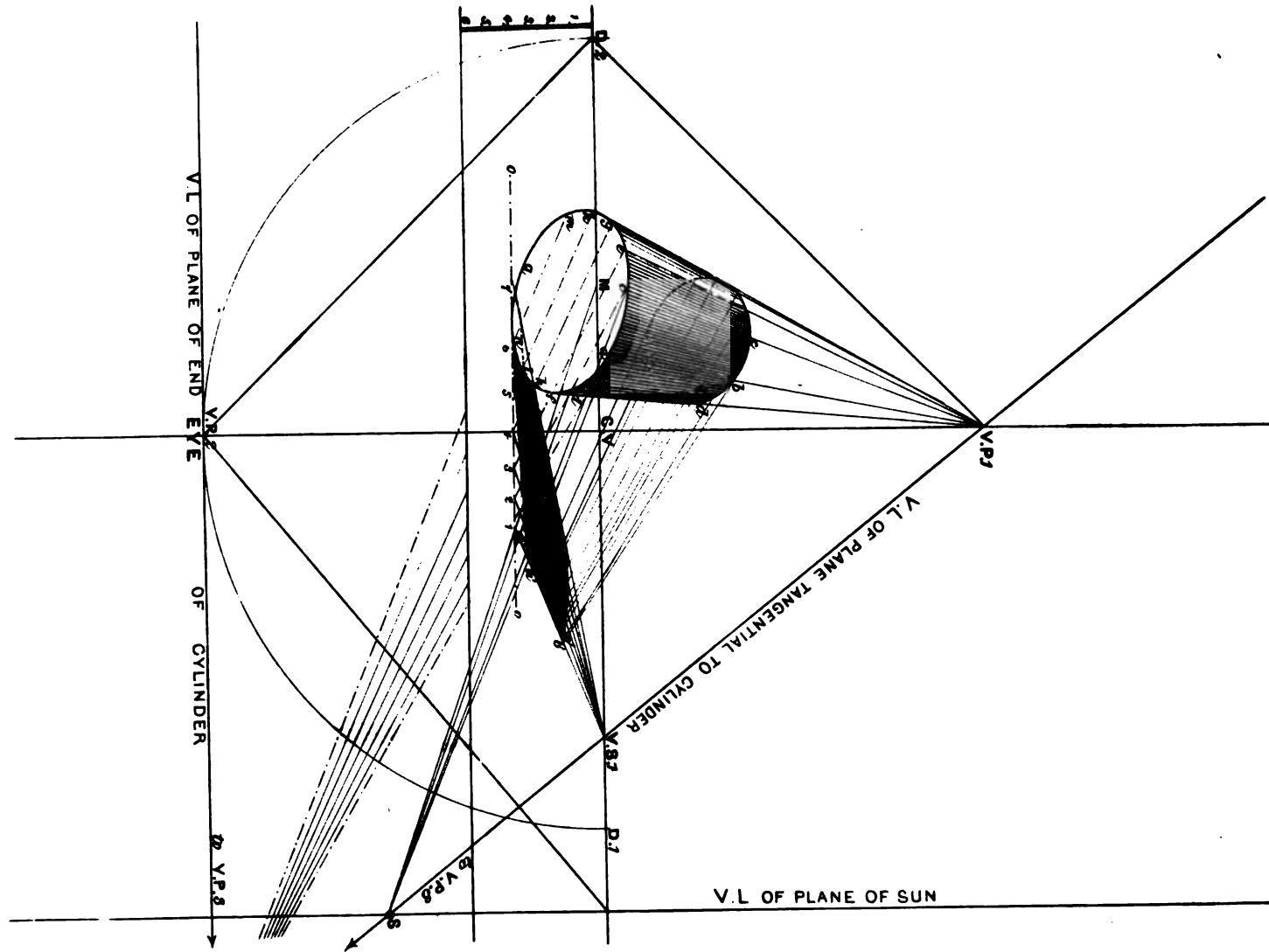


PLATE 42.

Distance of eye in front of the picture-plane, 18'.

Height of eye above the ground-plane, 6'.

A cylinder 15' long, 10' 6" diameter, touches the ground-plane at a point 7' 6" on the spectator's left and 10' beyond the picture-plane. The ends lie in planes which *descend directly* from the picture and spectator at an angle of 45° with the ground-plane.

Required the shadow of the cylinder upon the ground-plane, when the sun is in front of the picture-plane, on the left of spectator, lying in a vertical plane which makes an angle of 40° with the picture-plane. Altitude of sun's rays 20°.

Having obtained $\odot S$, the position of the sun, a tangential plane to cylinder passing through $\odot S$ must next be found.

Join $\odot S$ to $VP1$ (the vanishing point of the long edges of cylinder) and produce the line to meet the VL of plane of end of cylinder at $VP3$.

$VP3$ is the vanishing point of the intersection of the tangent plane with the plane of end of cylinder.

$VS1$ is the vanishing point of the intersection of the tangent plane with the shadow plane (ground).

The tangent plane cuts the plane of the end of cylinder in the line $a1$ drawn to $VP3$, it also cuts the plane of shadow in the line drawn from 1 to $VS1$, and its line of contact with the side of cylinder is shown by the line $a b$.

Then, since the tangential plane to the cylinder cuts the ground in the line 1, $VS1$, the shadow of $a b$ must lie somewhere upon this line; we have only to draw the rays of light from a and b to $\odot S$, meeting the shadow plane at a' and b' .

It is now necessary to assume any number of planes parallel to the *tangential plane* passing through $\odot S$ (the greater the number the more accurately the shadow will be defined) and find their intersections with the plane of shadow, and that of end of cylinder.

The intersections of these planes with the end of the cylinder are shown by the chain-lines $2 d e$, $3 f e$, $4 h g$, $5 l k$, $6 n m$, $7 q$, and these lines being actually parallel one to the other, are represented vanishing to $VP3$, *their common vanishing point*.

Draw the rays of light from e and d at both ends of cylinder to $\odot S$, these rays meet the ground on the line $2 VS1$ at $e' d'$, which are the shadows of e and d .

N.B. The points e and d at farther end of cylinder are found by joining e, d , at near end to $VP1$.

It is considered quite unnecessary to add any further explanation, as the remainder of the work is simply a repetition of that described for the points e and d .

It should be observed that the ends of the cylinder cast shadows upon the ground in the form of ellipses, portions of which are not really visible; however, the student will find them shown in the plate by dotted lines.

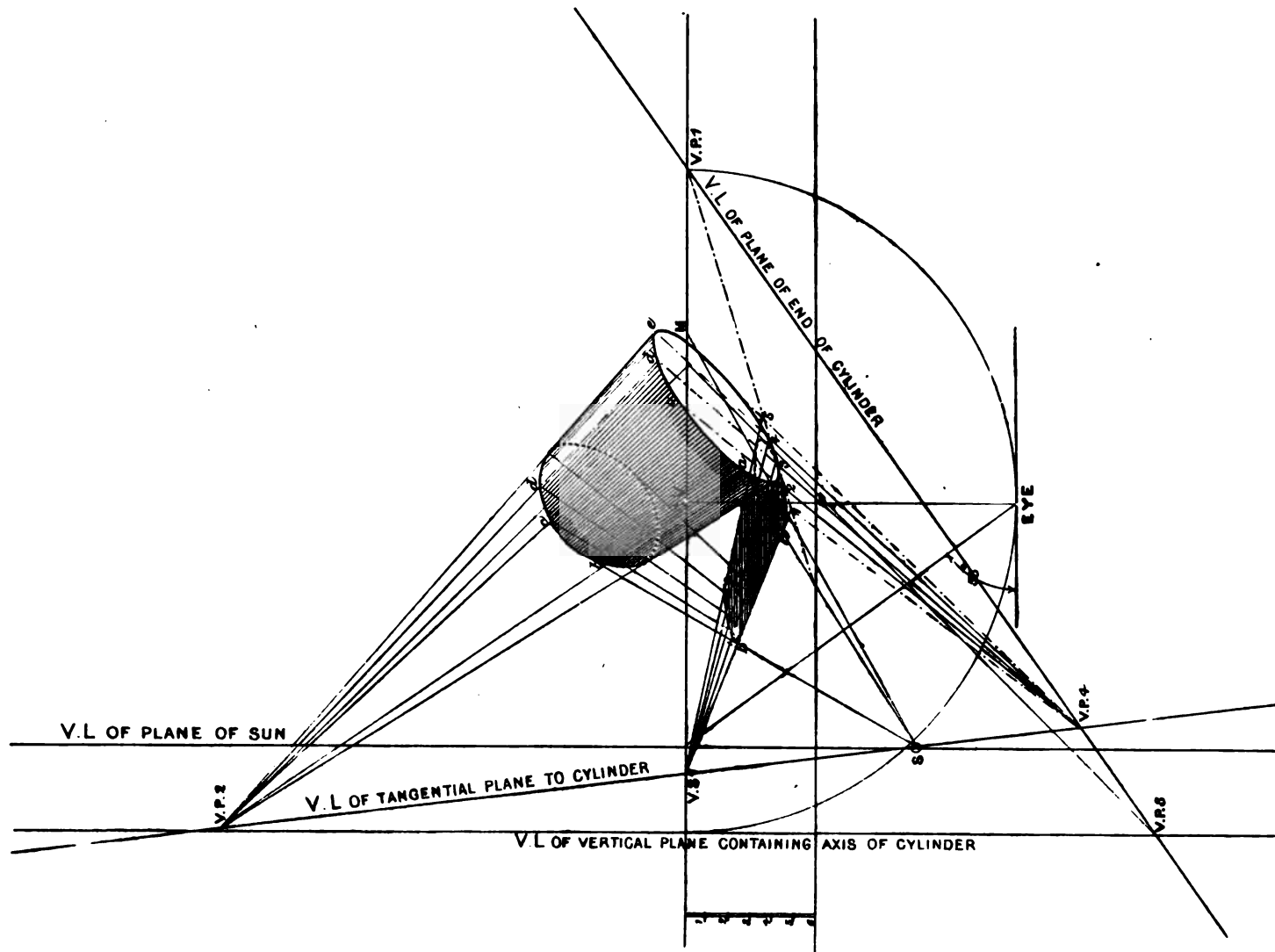


PLATE 43.

Distance of eye in front of the picture-plane, 15'.

Height of eye above the ground-plane, 6'.

A right cylinder 12' long, 10' 6" diameter, touches the ground at a point 3' on right of spectator, and 7' beyond the picture-plane. The ends of the cylinder lie in *oblique descending planes*, the direction of which with the picture is 45° towards spectator's right, and the inclination to ground 45°.

Required the shadow of the cylinder upon the ground when the sun is in front of the picture-plane, on the right of spectator, and the rays of light lie in vertical planes which make 55° with the plane of the picture. Sun's altitude 30°.

Find the position of the sun, $\odot S$, through it and $VP\ 2$ draw the

VL of a tangential plane to the cylinder, and produce it to meet the VL of plane of end of cylinder at $VP\ 4$.

The tangential plane to cylinder cuts the shadow plane (ground) in the line 1, $VS\ 1$; it also cuts the plane of the end of cylinder in the chain-line a, $VP\ 4$; and its line of contact with the side of cylinder is the line a b.

A series of planes parallel to the tangential plane passing through $\odot S$ should be now assumed.

These planes cut the ground in the lines 2, $VS\ 1$; 3, $VS\ 1$, &c. they also cut the plane of the end of cylinder in the chain-line c 2, d 3, &c.

It is presumed that the student will be able to complete the problem, since the remainder of the work is precisely that which has been fully described for the figures in preceding plates.

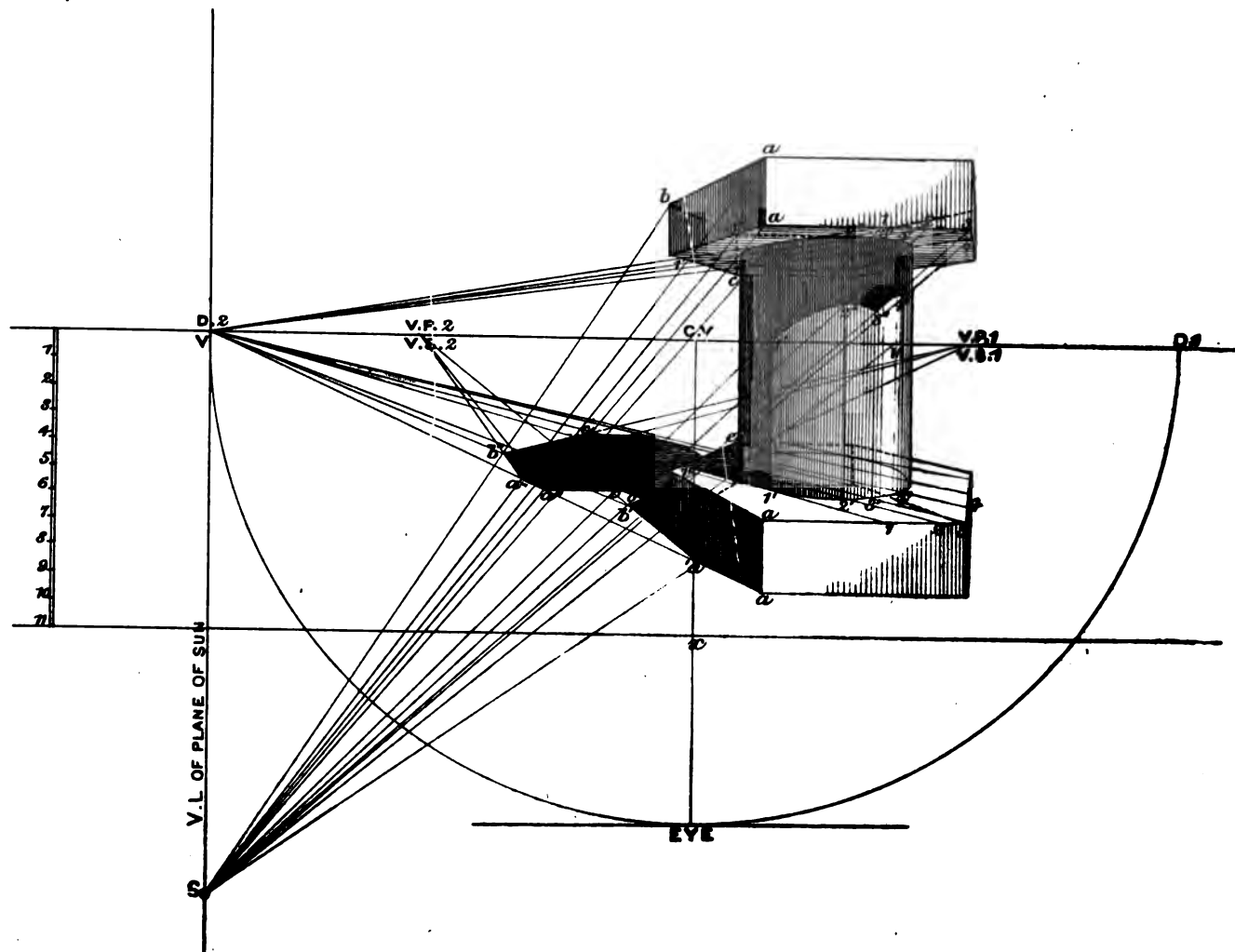


PLATE 44.

Distance of eye in front of the picture-plane, 18'.

Height of eye above the ground-plane, 11'.

I. A hexagonal slab of 9' edge, and 3' thickness, rests upon the ground-plane on one of its hexagonal faces, two edges of which are parallel to the picture-plane, and its nearest corner is at 3' on right of spectator, and 3' beyond the picture-plane.

II. A right cylinder, 10' diameter, 13' high, stands in the centre of the upper face of the hexagonal slab.

III. Upon the upper end of cylinder rests another hexagonal slab, of same dimensions, having its sides and corners immediately over those of the first hexagonal slab.

IV. Required the cast shadows of these solids upon the ground-plane and upon each other. The sun to be in front of the picture, on right of spectator, and the rays of light to lie in vertical planes making 45° with the picture. Sun's altitude 40°.

Determine $\odot S$ the position of the sun, then imagine a vertical plane to pass through it and the near upright edge, $a a$, of each hexagonal slab. The intersection of this vertical plane with the ground is the line drawn from a to V ($D 2$), and since the rays of light drawn

from corners a, a, a , are supposed to lie in the vertical plane they must evidently meet the ground on line $a V$, at points a', a'', a''' .

$a a'$ is the shadow of near edge of lower slab, and $a' a''$ represents the shadow of the near edge of upper slab.

N.B. The edges forming the hexagonal faces of the slabs are parallel to the plane of shadow (ground), therefore the edges and their shadows must have the same vanishing points. (See general rule C.)

Join a', a'' to $V S 2$, then draw the rays of light $b b', b b''$, which will determine $a' b', a'' b''$, the shadows of the edges $a b$ of upper and lower slab.

Join b' to $V S 1$ ($V P 1$) a portion of the shadow of $b c$ of *lower slab* will be seen on this line, the remainder being hidden by the shadow of the cylinder.

Also join b'' to $V S 1$; then draw a ray of light from corner c (upper slab) which determines the length of the shadow $b'' c$.

The remainder of the shadow of upper slab *upon the ground* is simply a repetition of the work described for points a, b, c ; and since all the construction lines are shown it is presumed that the student will not require further explanation.

To obtain the shadow of the cylinder upon the upper surface of the lower hexagonal slab and the ground-plane, it is necessary to

imagine a vertical plane passing through $\odot S$ tangential to the cylinder. Its line of contact with the cylinder is represented by the line $1', 1'$. Its intersection with the upper surface of the lower slab is the line $1, 1'$, drawn through $1'$ to V . It cuts through the thickness of the slab in the vertical line $1'' 1''$; it also intersects the ground-plane in the line $1'' V$ (D 2).

The line $1', 1'$, is the shadow of the near side of cylinder upon the *upper surface of lower slab*, and its continuation upon the ground is seen upon the line $1'' V$ between the points O, O .

We must now imagine a series of vertical planes to pass through the sun and cut the cylinder and slabs; these planes will be parallel to the tangential plane, and have their intersections with the slabs vanishing to V .

The lines $2 V, 3 V, 4 V$, shown upon the hexagonal surfaces of the slabs, represent the intersections with those surfaces of the vertical section planes.

For example, the first vertical section plane has its intersection with the slabs in the lines $2 V, 2 V$; these intersections meet the curved lines of ends of cylinder at points $2', 2'$, therefore, if $2' 2'$

be joined by a vertical line we shall have the intersection of the first vertical plane of section with the convex surface of the cylinder.

Draw a ray of light from point 2 , *upper slab*, which is supposed to lie in the plane of section, and meet the cylinder upon the line $2', 2'$, at point $2''$. Point $2''$ is the shadow of 2 .

The shadow of point 1 , *upper slab*, is determined by drawing a ray of light from it, in tangential plane to cylinder, to meet the line of contact at $1'$.

The second vertical plane of section is taken through the corners $3, 3$ of the slabs. Its intersection with the convex surface of the cylinder is represented by the line $3', 3'$, and the ray of light drawn from 3 , *upper slab*, meets the cylinder at $3''$, which is another point in the required shadow.

It will be observed that the shadow of the straight line $a 3$, *upper slab*, upon the cylinder is the curved line drawn through the points $1', 2'', 3''$.

Any number of points in the shadow may be determined upon the same principle as $2'', 3''$.

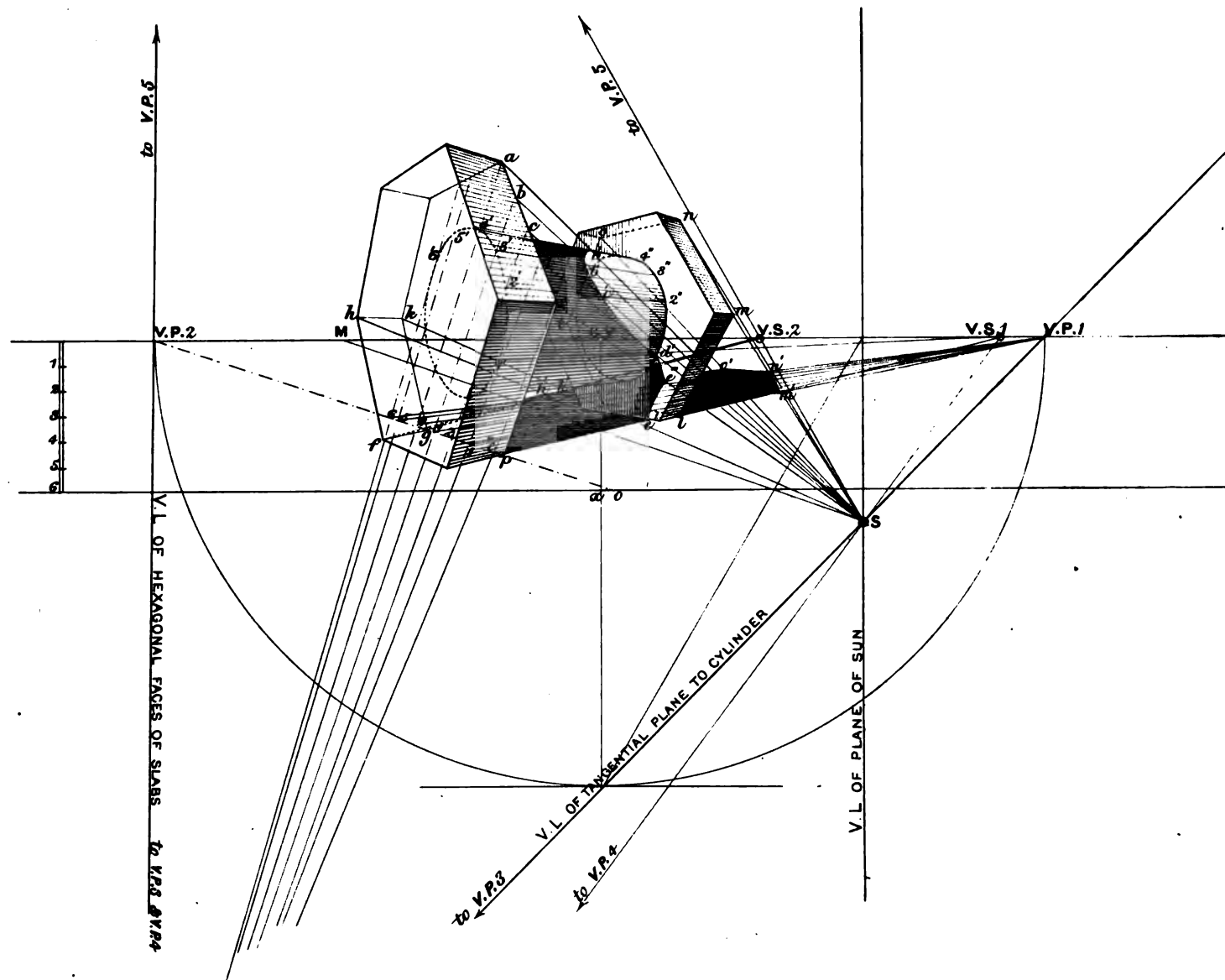


PLATE 45.

Distance of eye in front of the picture-plane, 18'.

Height of eye above the ground-plane, 6'.

Required the perspective representation of the three solids, *preceding plate*, when each slab rests upon the ground on one of its rectangular sides, the hexagonal faces of the slabs are vertical at an angle of 45° with the picture-plane towards spectator's left. The nearest corner of *first* hexagonal slab touches the plane of the picture.

Required the cast shadow of the first hexagonal slab upon the other solids and the ground-plane. The sun being in front of the picture-plane, on left of spectator, in a vertical plane, making 60° with the plane of the picture. Sun's altitude 20° .

Obtain $\odot S$, the position of the sun.

It is advisable to find the shadow of the line $p e$ (first slab) upon the ground, and nearest vertical face of the second slab.

Join $V P 4$ to $\odot S$, produce the line to meet $V L$ of ground (horizon) at $V S 1$, which is the vanishing point of that portion of the shadow of line $p e$ which falls upon the ground.

Join p to $V S 1$ meeting lower edge of second slab at e' ; then, because $p e$ is parallel to the inner vertical surface of second slab, the remaining portion of the shadow ($e' e''$) must be drawn to $V P 4$, the length of which is determined by the ray of light $e e''$.

The line $a e$ casts a shadow upon the convex surface of cylinder, likewise upon the second slab.

A portion of the line ($d e$) casts a shadow upon the second slab; and, because it and the slab are parallel, the line and its shadow have same vanishing point.

Draw a line from e'' to $V P 5$, then draw a ray of light from d to $\odot S$; the ray meets the former line upon the surface of the slab at d' , and determines the shadow.

Now find the $V L$ of a plane passing through the sun tangential to the convex surface of the cylinder.

Join $\odot S$ to $V P 1$, and produce the line to meet the $V L$ of the planes containing the hexagonal faces of the slabs at $V P 3$.

$V P 3$ is the vanishing point of the intersection of the tangent plane to cylinder with the hexagonal surfaces of the slabs.

$V P 1$ is the vanishing point of the intersection of the tangential plane to cylinder with the ground-plane.

In order to find the shadow of the remaining portion of the line $a e$ upon the convex surface of cylinder the student must imagine a series of *section planes* (planes cutting through the solids) parallel to tangent plane passing through the sun.

These imaginary section planes intersect the inner surface of the *first* hexagonal slab in the chain-lines $a 4$, $b 3$, $c 2$, $d 1$, $e e''$, &c., and

meet the ground plane in the lines drawn from e' , 1, 2, 3, 4, 5, 6, to $V P 1$.

The section planes also meet the convex surface of the cylinder in the right lines $4' 4''$, $3' 3''$, $2' 2''$. The tangent plane to cylinder has its line of contact with the near side of cylinder in the line $1' d'$.

In these section planes the rays of light are supposed to lie; therefore they and the section planes containing them, must meet the cylinder, the section planes in right lines, and the rays in points upon these right lines.

For example, the chain-line $b 3$ is the intersection of one of the imaginary section planes with the inner surface of the *first* slab, its intersection with the convex surface of the cylinder is the right line $3' 3''$, and the ray of light $b \odot S$, *lying in this section plane*, meets the cylinder upon the right line $3' 3''$ at b' .

It is hoped that the student will find this explanation sufficiently lucid, as the shadows of the points a , c , d are determined by the method described for point b .

Having found a' , b' , c' , d' we have simply to join them by a curved line which represents the shadow of the portion $a d$ of the right line $a e$ upon the surface of the cylinder. The shadow of the *second* slab upon the ground-plane should be now determined.

Join $V P 4$ (the vanishing point of the line $l m$) to $\odot S$ and produce the line to meet $V L$ of ground-plane at $V S 1$.

The line joining $V P 4$ to $V S 1$ is really the $V L$ of a plane passing through the sun containing the line $l m$; and the shadow

of $l m$ will lie on the intersection of this plane with the plane of shadow (ground-plane).

Join l to $V S 1$, then draw the ray of light $m \odot S$ to meet former line at m' , giving $l m'$ as the shadow of right line $l m$.

We next join the sun to $V P 5$ (vanishing point of right line $m n$) and the vanishing point of its shadow will be obtained at point $V S 2$.

Join m' to $V S 2$, a ray of light drawn from n to meet the ground at n' gives the length of the shadow.

Then, since the line $n o$ (top of back surface of *second* slab) is parallel to the plane of shadow we have simply to join n' to $V P 2$ and draw the ray of light $o o'$.

Only a small portion of the shadow of the line $o t$ is seen by the spectator, and since the original line $o t$ is parallel to $l m$ its shadow must obviously have the same vanishing point; viz., $V S 1$.

Join O' to $V S 1$.

A portion of the shadow of the line $f h$ is seen; $V S 2$ is its vanishing point, because $f h$ is parallel to $m n$.

Join f to $V S 2$, and the ray of light $h h'$ will determine the length of the shadow.

The line $h k$ forming the thickness of *first* slab, is parallel to the plane of shadow, therefore draw its shadow, $h' k'$, to $V P 1$, and the remaining small portion of shadow will have $V S 1$ for its vanishing point.

N.B. The shadow of an abacus upon a column would be obtained by the method employed for the solids in this exercise.

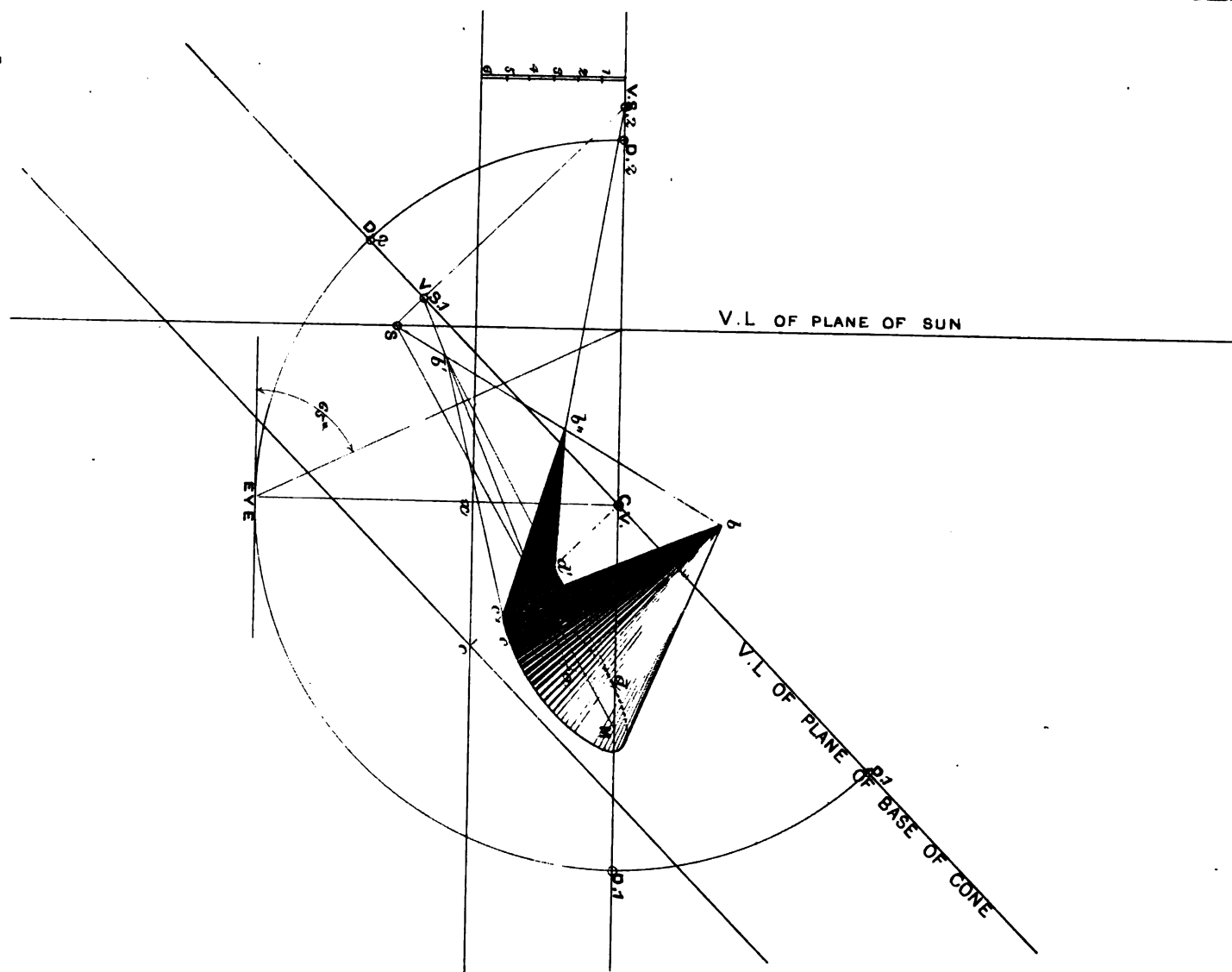


PLATE 46.

Distance of eye in front of the picture-plane, 15'.

Height of eye above the ground-plane, 6'.

A right cone 10' diameter, 12' high, has its base in an *ascending plane* perpendicular to the picture and inclined to the ground at an angle of 45° . The base touches the ground at a point 6' on right of spectator and 6' beyond the picture-plane.

Required the shadow of the cone upon the ascending plane of the base and the ground-plane. The sun is to be in front of the picture-plane, on the right of spectator, and the rays of light in vertical planes making 65° with the picture. Sun's altitude, 30° .

First obtain the shadow of the axis of the cone upon the ascending plane of the base produced.

It should be observed that the line *a b* has no vanishing point, we must therefore draw through $\odot S$ parallel to it, and produce the line to meet the *V L* of plane of base at *V S 1*, also the *V L* of ground-plane at *V S 2*.

V S 1 is the vanishing point of the shadow of *a b* upon the

ascending-plane of base, and V S 2 that of its shadow upon the ground-plane.

Join *a* to *V S 1*, then draw a ray of light from the apex of the cone to $\odot S$ meeting line *a V S 1* at *b'*, which is the shadow of the apex on the plane of base produced.

From *b'* draw tangents *b' c*, *b' d*, to the base of the cone; points *c* and *d* are their points of contact.

The shadow of the triangular plane c b d will be the required shadow of cone.

b' c d is the shadow upon the ascending-plane of base produced.

The intersection of the ground-plane with the ascending-plane of base is represented by the chain-line *C V O*, and the shadow *b' c d* intersects this line at *c' d'*.

The shadow of the axis meets *C V O* at point *a'*, therefore, if we join *a'* to *V S 2*, and draw a ray of light from the apex of cone to meet it at *b''*, we shall have the shadow of the apex upon the ground-plane.

Finally, join *c' b''*, *d' b''*, which completes the shadow upon the two planes.

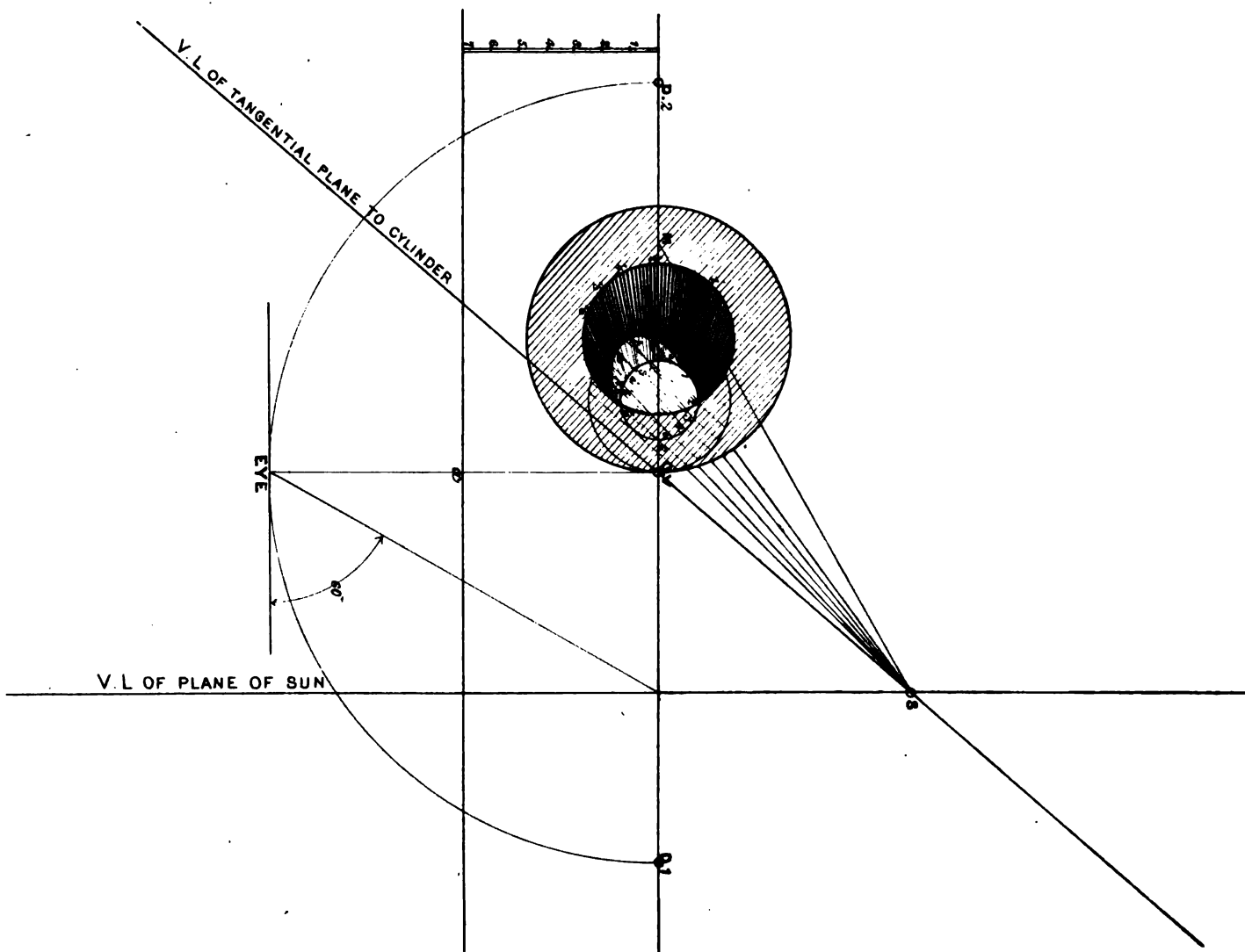


PLATE 47.

Distance of eye in front of the picture-plane, 16'.

Height of eye above the ground-plane, 7'.

Draw in perspective a cylindrical tube 20' long, 14' external diameter, 3' thick. It lies upon its side and its line of contact with the ground-plane is perpendicular to the picture-plane, 7' on left of spectator, its near extremity being 8' beyond the picture-plane.

Required the shadow of the curved line of farther end upon the interior concave surface of the tube, when the sun lies behind the picture-plane, on right of the spectator, in a vertical plane which makes 60° with the picture. Sun's altitude 30° .

Obtain the **V L** of a plane passing through the sun tangential to the inner surface of the tube.

This **V L** is determined by joining **⊙ S** to **C V**.

The line **1, 1'** represents the line of contact of the tangent plane with the surface of the tube, it is the deepest part of the shadow, and point **1** is the commencement of the shadow upon the interior surface at farther end of tube.

By employing a series of imaginary *section planes* parallel to the tangent plane passing through the sun, we shall be enabled to find other points in the required shadow.

The chain-lines **22', 33', 44', 55', 66'**, drawn across the farther end of cylindrical tube, represent the intersections of the section planes with that end, while the right lines **2' 2'', 3' 3'', 4' 4'', 5' 5'', 6' 6''**, drawn to **C V**, represent the intersections of the section planes with the concave surface of the tube.

A ray of light must be imagined to lie in each section plane, and the points of intersection of these rays with the concave surface of the tube will be points in the required shadow.

For example, the section plane which has its intersection with farther end of tube in the chain-line **2 2'** cuts the inner surface of the tube in the line **2' 2''**; then, if a ray of light be drawn through **2** to **⊙ S**, it will be found to meet the inner surface of the tube also upon the line **2' 2''** at point **a**.

Obtain points **b, c, d, e** by the method described for point **a**. A curved line drawn through these points is the shadow required.

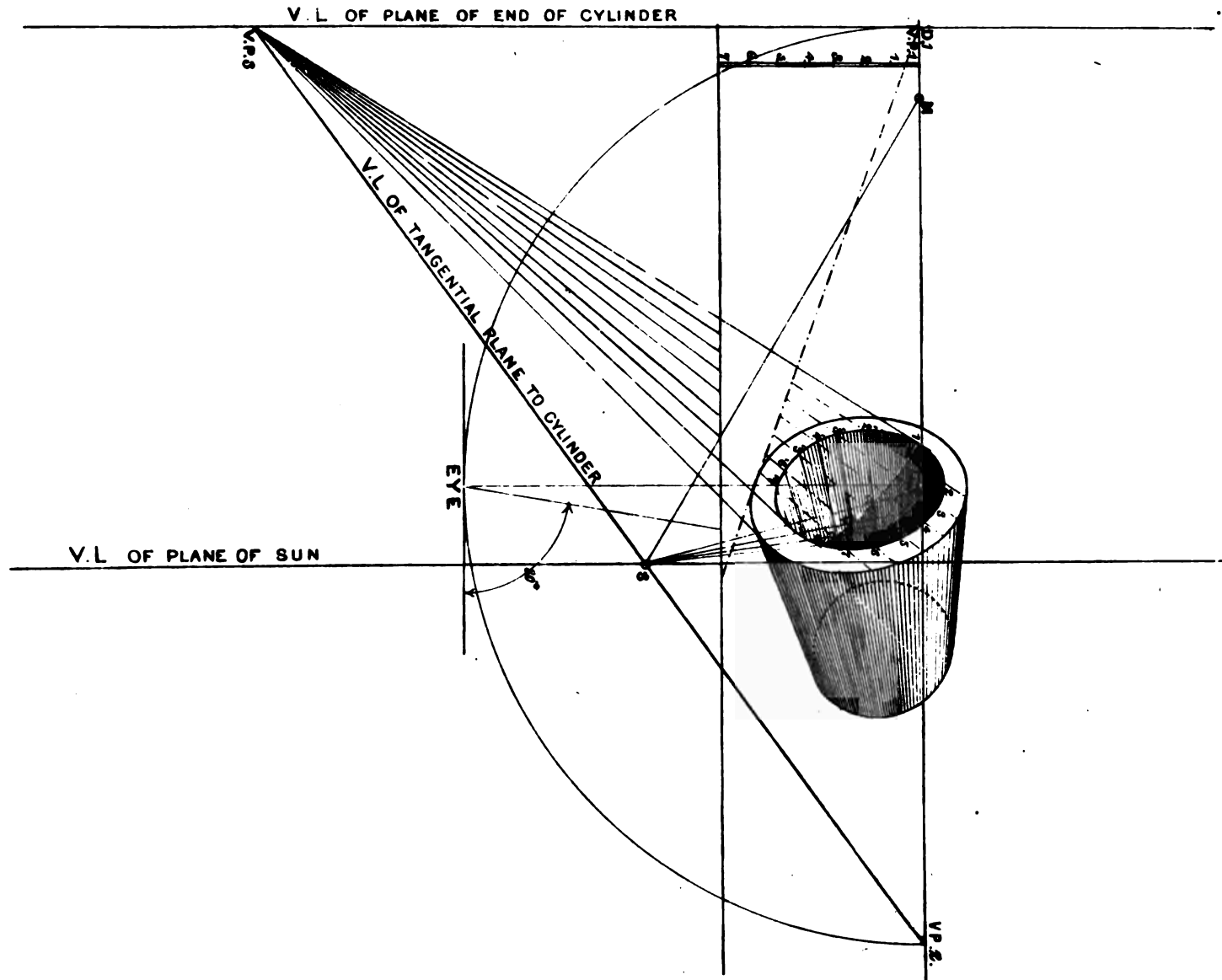


PLATE 48.

Distance of eye in front of the picture-plane, 16'.

Height of eye above the ground-plane, 7'.

Draw in perspective a cylindrical tube 14' long, 9' external diameter, and thickness 1'. It lies upon the ground-plane upon its side, having its ends in vertical planes making 45° with the picture-plane towards the left of the spectator. The near end touches the plane of the picture at a point 3' on spectator's right.

Required the shadow of the curved line of near end upon the inner surface of the tube, when the sun is in front of the picture, on the left of the spectator, and the rays of light are in planes making 80° with the picture. Sun's altitude 32° .

Obtain the **V L** of a plane tangential to the inner surface of the cylindrical tube passing through the sun.

Join $\odot S$ to **V P 2** and produce the line to meet the **V L** of plane of end of tube at **V P 3**.

V P 2 is the vanishing point of the line of contact of tangent plane with the tube.

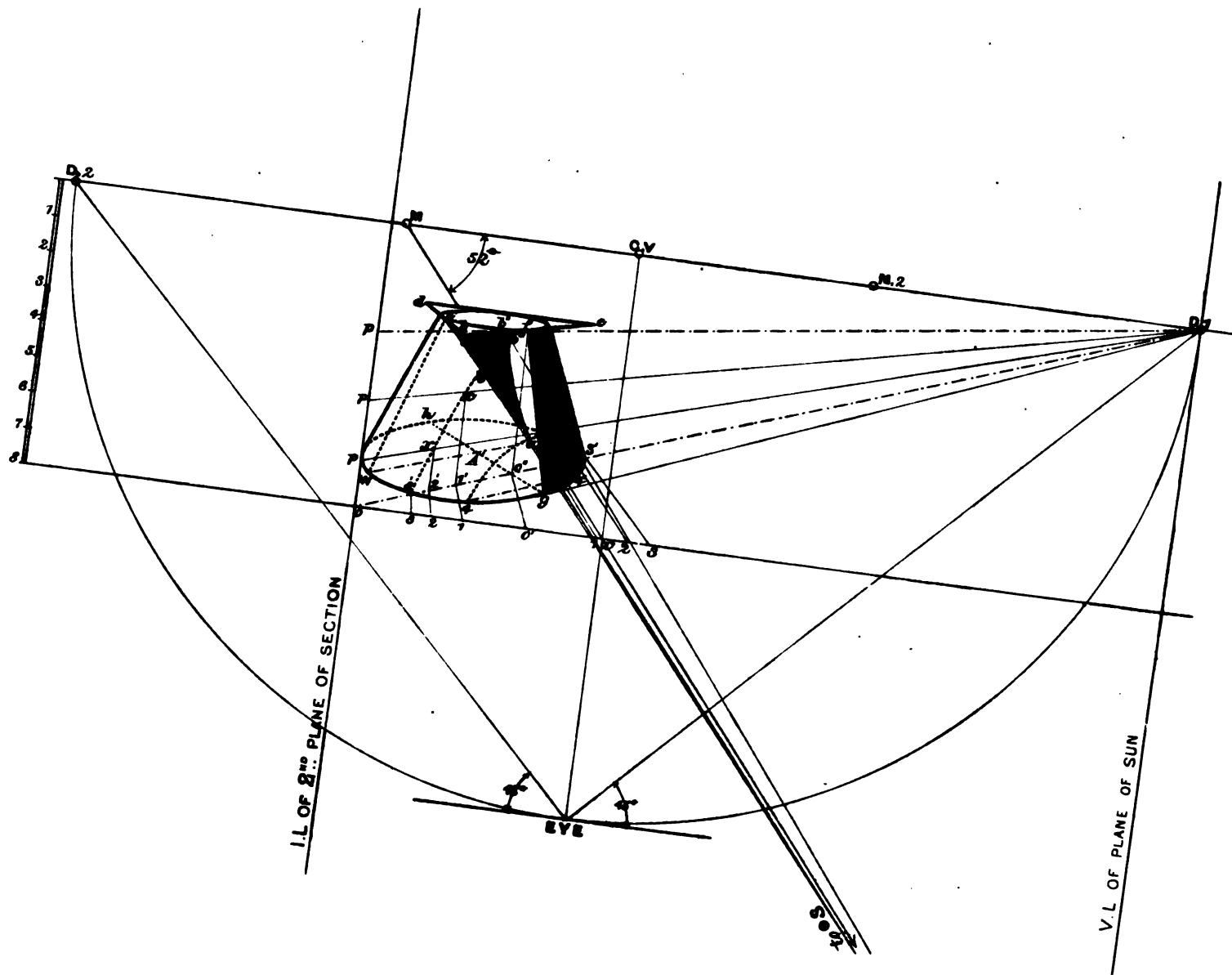
V P 3 is the vanishing point of the intersection of the tangent plane with the plane of the end of tube.

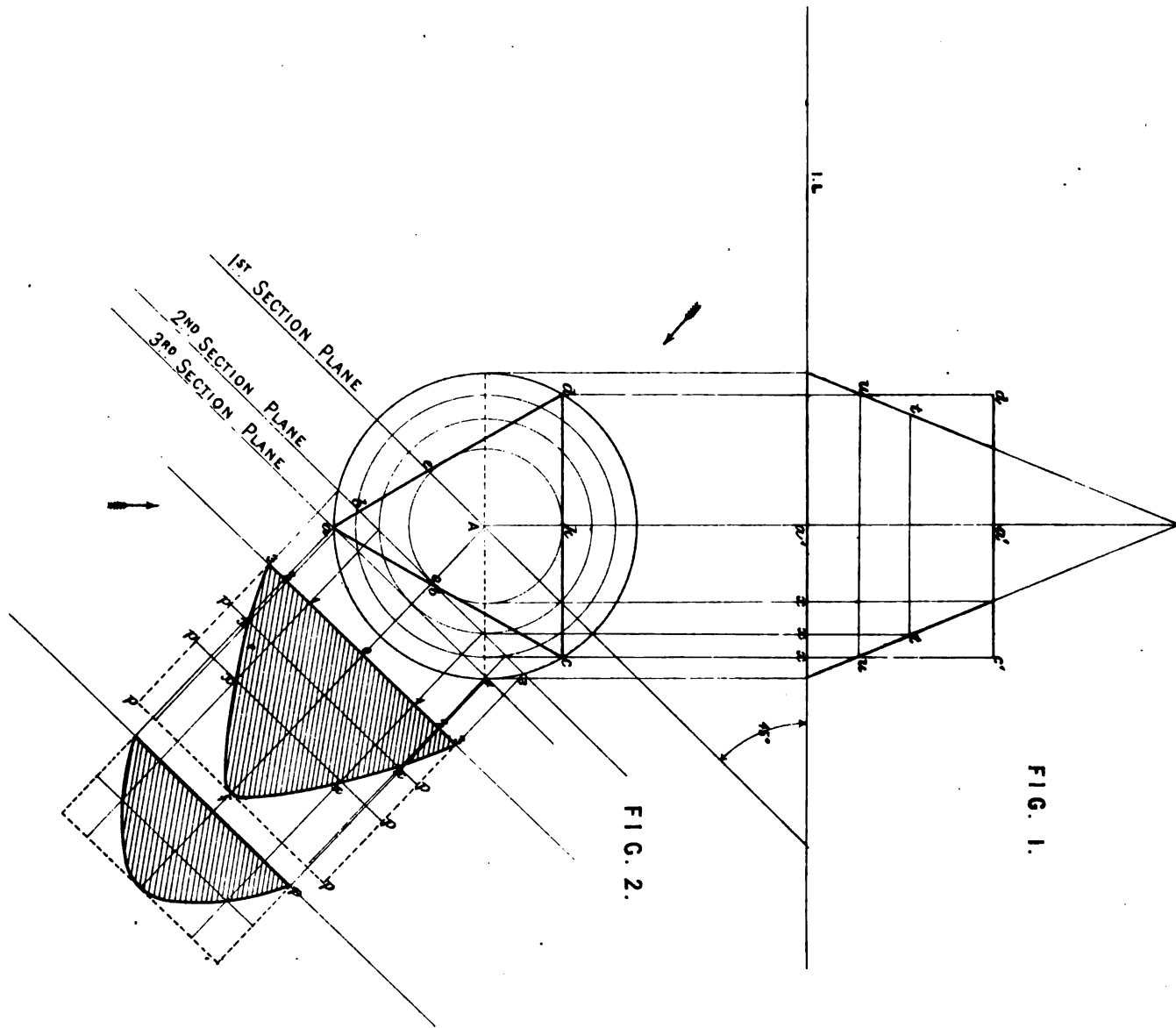
As in the previous exercises, we must imagine a series of section planes parallel to the tangent plane passing through $\odot S$.

The intersections of these section planes with the near end of tube are shown by the chain-lines 2 2', 3 3', 4 4', 5 5', 6 6', 7 7', and the right lines drawn from 2', 3', 4', 5', 6', 7', represent the intersections of the same planes with the concave surface of the cylindrical tube.

Finally, draw the rays of light 2 $\odot S$, 3 $\odot S$, 4 $\odot S$, 5 $\odot S$, &c., to meet the inner surface of tube in their respective section planes at points a, b, c, d, and the curved line of shadow must be drawn through these points, commencing at point 1.

N.B. The shadow of the intrados of an arch upon the inner surface of an archway would be determined by the methods described for this and previous exercises.





PLATES 49, 50.

Distance of eye in front of the picture-plane, 16'.

Height of eye above the ground-plane, 8'.

I. A right cone 8' diameter at base, 10' high, is cut by a horizontal plane at half its height. Show the lower portion of the cone when its axis is vertical at 5' on left of the spectator, and 5' beyond the picture-plane.

II. Upon the horizontal section of the cone rests an equilateral triangular plane, its centre coincides with that of the cone, and its edges are tangential to the circular section of the cone. The edge of the triangular plane farthest from the spectator is parallel to the picture-plane.

III. Required the shadow of the equilateral triangular plane upon the convex surface of the cone, when the sun is in front of the picture-plane, on the left of spectator, and the rays of light make 45° with the picture, and 52° with the ground-plane.

The shadow of the triangular plane upon the surface of the cone will be determined by means of a series of section planes; it is therefore absolutely necessary that the student should know how to find these sections.

Figs. 1 and 2, plate 50, represent the elevation and plan of the cone and triangular plane.

The sun lies in a vertical-plane which makes 45° with the picture, therefore the imaginary vertical sections of the cone and triangular plane must make the same angle.

Imagine the line drawn through $e A$ at an angle of 45° with $I L$ to be the plan of the *first* vertical section-plane passing through the centre of the cone.

By viewing the section at right angles to $e A$ we shall obtain its *true form*, which is manifestly that of the elevation of the cone, fig. 1.

Suppose a *second* vertical section-plane parallel to first to pass through the cone on the line 303, fig. 2.

We have now to determine the *true form* of section by viewing the object at right angles to this section-plane, in the direction of the arrow O .

The line **33** is the section of the plane surface of the base of the cone of its actual length. Point O , on the line **33**, is the plan of the highest point of the section; its actual distance from the ground is equal to the length of the line $a' a'$ (fig. 1), therefore make $o f$ (fig. 2) equal to $a' a'$.

Now take any number of *horizontal sections* between $a' a'$ as $t t$, $u u$, fig. 1.

The plans of these imaginary horizontal sections are shown by the two concentric circles which cut the line **303** in points 1, 2, on either side of O , fig. 2.

Draw $2 x$, $2 x$, on either side of O , parallel to $o f$, and equal in length to $x u$, fig. 1, also draw $1 x$, $1 x$, parallel to $o f$ and equal in length to $x t$, fig. 1.

The true form of the imaginary second vertical section is the curved line (*hyperbola*) drawn through the points 3, x , x , f , x , x , 3, fig. 2.

N.B. Any number of imaginary vertical sections may be made, and their true forms ascertained by the method described for the *second section*.

TO FIND THE PERSPECTIVE REPRESENTATION OF THE VERTICAL SECTIONS
OF THE CONE.

Draw a line, *g h*, through the centre of the base of the cone, *A'*, *perpendicular* to the vertical section planes; *D 2* is its vanishing point.

Through *A'* draw a chain-line to *D 1*, which represents the intersection of *first* section plane with the base of cone; the dotted line *e w* is its intersection with the near surface of the cone.

The shadow of the triangular-plane upon the surface of the cone will commence at point *e* and terminate at point *f*.

Measure from *A'*, on line *g h*, a distance *A' O''* perspectively equal to *A o*, fig. 2, plate 50. *M 2* is the measuring point by which the perspective length *A' o''* is obtained.

Through *O''* draw the intersection with the ground of the *second* section plane which is supposed to pass through the sun.

Produce this intersection (*chain-line drawn through O'' to D 1*) to meet the picture-plane at *O*. Through *O* draw the *I L* of second section plane.

Bring forward *O''* by *M 1* to meet the picture-plane at *O'*, on either side of it set off the distances *O' 1*, *O' 2*, *O' 3*, equal to *O 1*, *O 2*, *O 3*, fig. 2; then join these points to *M*, giving points *1'*, *2'*, *3'*, on either side of *O'* in perspective.

Now set up the distances *o p*, *p p*, *p p*, on *I L* of second plane of section, equal to *3 p*, *p p*, *p p*, fig. 2. Join the points *p* to *D 1*, and to meet these lines draw verticals from points *2'*, *1'*, *O'*, *1'*, *2'*, giving the four points *x*, and the highest point of the section, *f*.

The dotted curved line drawn through *3'*, *x*, *x*, *f*, *x*, *x*, *3'*, is the perspective representation of the section made by the second vertical section-plane.

The line which separates the light from shadow on the surface of the cone is determined by joining *f g*.

The student should now refer to fig. 2, plate 50, and he will perceive that the *second* section plane cuts a piece off the triangular plane *b b a*.

The points *b b* are situated in the section plane, therefore their perspective representations must be determined upon the intersection of the second vertical plane of section with the triangular plane.

The chain-line *p, D 1* is the intersection above mentioned, it cuts the near sides of the triangular plane at points *b*, *b''*.

Draw rays of light from *b*, *b''* to *⊙ S*, lying in second plane of section, meeting the convex surface of the cone upon the curved line of section at points *b'*, *b'''*. *b'*, *b'''* are two points in the required shadow.

The *third* vertical plane of section is supposed to pass through the sun, and *a 4* fig. 2, plate 50. The true form of this section, and its perspective representation are shown, and have been obtained by the method described for second section.

Since the third vertical plane of section contains the near corner (a) of the triangular plane, it is manifest that the shadow of *a* will be determined upon the contour of this section.

Draw a ray of light from *a* to *⊙ S* which comes into contact with the surface of the cone upon the curved line of section at point *a'*.

Join *e*, *b'*, *a'*, and *a'*, *b'''*, *f*, for the shadow of the triangular plane upon the convex surface of the cone.

N.B. The shadow of a vase would be determined by the methods described in this and previous exercises.

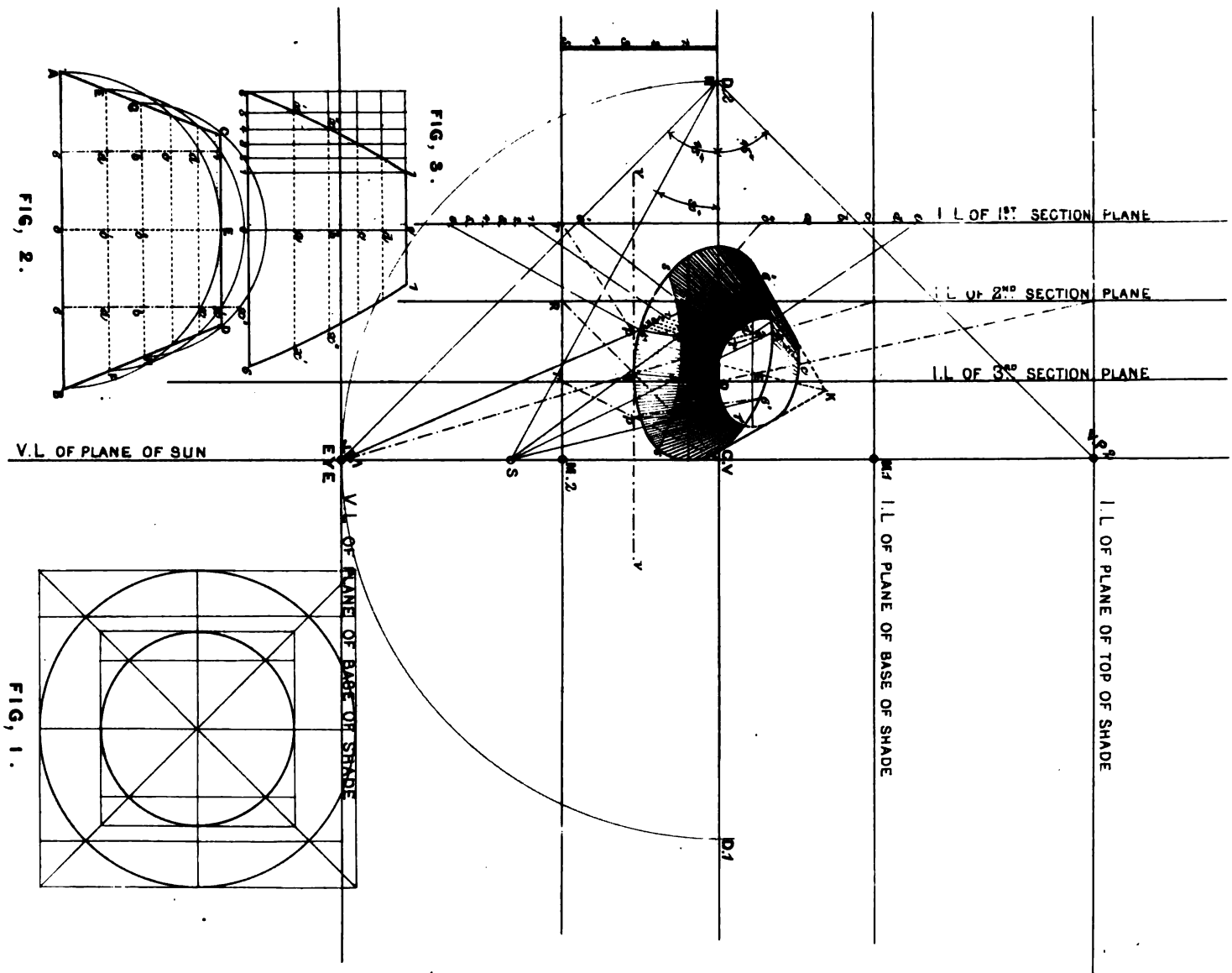


PLATE 51.

Distance of eye in front of the picture-plane, 12".

Height of eye above the ground-plane, 5".

A conical lamp shade is 5" high, diameter of upper end 6", diameter of base 10". Give its perspective representation when the base touches the ground at a point 5" on the left of spectator and 10" beyond the picture-plane. The plane of the base *descends directly* from the picture-plane at an angle of 45° with the ground-plane.

Required the shadow of the curved line of the base upon the interior concave surface of the lamp-shade, when the sun is in front of the picture-plane at an altitude of 30°, and the rays of light lie in vertical planes perpendicular to the plane of the picture.

By referring to plate 27 the student should experience no impediment in obtaining the perspective representation of the lamp-shade.

In order to obtain the perspective representation of the required shadow we must imagine a series of vertical planes to pass through the sun and cut the lamp-shade, each plane to be perpendicular to the plane of the picture, and the true form of each section must be determined as in preceding plate.

Figs. 1 and 2 are the plan and elevation of the conical lamp-shade, and the lines 1 6, 0 E, 1 6, fig. 2, are the elevations of the ima-

ginary vertical section planes, the true forms of sections made by these planes we now proceed to obtain.

The vertical section-plane 0 E passes through the centre of the lamp-shade, consequently the true form of section must be the same shape and size as the elevation A B C D, fig. 2.

It should be observed that the other vertical planes of section 16, 16, are equally distant from the axis 0 E, therefore the true form of each will be alike, and *one* only need be described.

Take any number of imaginary horizontal sections of the lamp-shade, as shown by the dotted lines drawn through a, b, c, d, fig. 2, then if the distance from each of these points to the opposite side of the lamp-shade be ascertained, the true form of the section will be easily obtained.

For example, a semicircle described from 0 as centre (fig. 2) upon the line A B represents *one half* of the base of the lamp-shade of its actual form, then it is very evident that the distance from point 6 right through the base must be *twice* the length of the line 6 x.

Make 0 6, 0 6, fig. 3 (base of true form of section) equal to 6 x, fig. 2.

Now describe a semicircle upon the line E F (fig. 2) from 0' as centre; this semicircle represents *one half* of the horizontal section

taken through point **a**, therefore the vertical distance **a x** represents *one half* the distance through the lamp-shade at point **a**.

Measure **6 a**, fig. 2, and make **0 a**, fig. 3, equal to it.

Then, upon the line drawn through point **a**, fig. 3, on either side of **a**, set off **a x'**, **a x'**, equal to **a x'**, fig. 2.

I propose showing how to find the distance through the lamp-shade at point **b**, after which the student should certainly be able to find those from **c**, **d**, **1**.

The horizontal section through **b**, fig. 2, is equal to the distance **0 0'** above the base line **A B**, therefore make **0 b**, fig. 3, equal to same.

A semicircle described from **0'**, fig. 2, upon the line **G H**, represents *one half* the actual form of horizontal section, therefore **b x''** (fig. 2) must represent *one half* the distance through the lamp-shade at point **b**.

Measure **b x''**, **b x''**, fig. 3, equal to **b x''**, fig. 2.

Points **x''**, **x''**, fig. 3, are two more points in the required true form of section through which draw the curved line of section.

TO FIND THE PERSPECTIVE REPRESENTATION OF THE SECTIONS OF THE LAMP-SHADE.

It will be advisable to commence by finding the *second* section-plane passing through the centre of lamp-shade.

Bring out **B** by **C V** to meet the picture-plane at **R**; through **R** draw the **I L** of second section-plane.

Set off **R r**, **R r**, on the ground-line equal to **0 6**, **0 6**, fig. 2, and through the points **r r**, draw the intersecting lines of the first and third section planes.

The chain-lines **r C V**, **R C V**, **r C V**, represent the intersections with the ground-plane of the three imaginary vertical planes of section, and upon careful observation it will be seen that these chain-lines come into contact with the intersection of the plane of base of lamp-shade with ground (*chain-line V V*) at points **p B p**.

The chain-lines **p 6'**, **B A**, **p 6'** are the intersections of the three vertical planes of section with the plane of the base of the lamp-shade.

Upon the line **6' 6''** lying in *first section plane*, obtain the perspective distances **0, 1, 2, 3, 4, 5**, by **M 1**, equal to the corresponding measurements **0, 1, 2, 3, 4, 5**, fig. 3.

Join **0** (*lying in first section-plane*) to **V P 2**, then by **M 2** obtain the distances **0, a', b', c', d', e'**, perspective equal to the corresponding distances **0, a, b, c, d, e**, fig. 3.

Join **a', b', c', d', e'** to **V P 1**, also join **1, 2, 3, 4, 5**, on the line **6' 6''** to **V P 2**, and where these two sets of lines intersect upon the concave surface of lamp-shade, the curved line of section should be drawn.

A ray of light from **6''**, lying in *first section-plane*, meets the line of section just obtained, at point **S**, which is a point in the required shadow.

N.B. *Only the lower half of each section need be obtained in the perspective representation.*

For the *second section* we have simply to draw **B D**, **A C**, to **K** (imaginary apex of cone), then draw the ray of light from **A** to **⊙ S**, giving **S**, another point in the required shadow upon the line **B D**.

The third section is found precisely in the same way as the first and does not require further explanation.

Having found the several points **S**, the outline of the required shadow should be drawn through them.

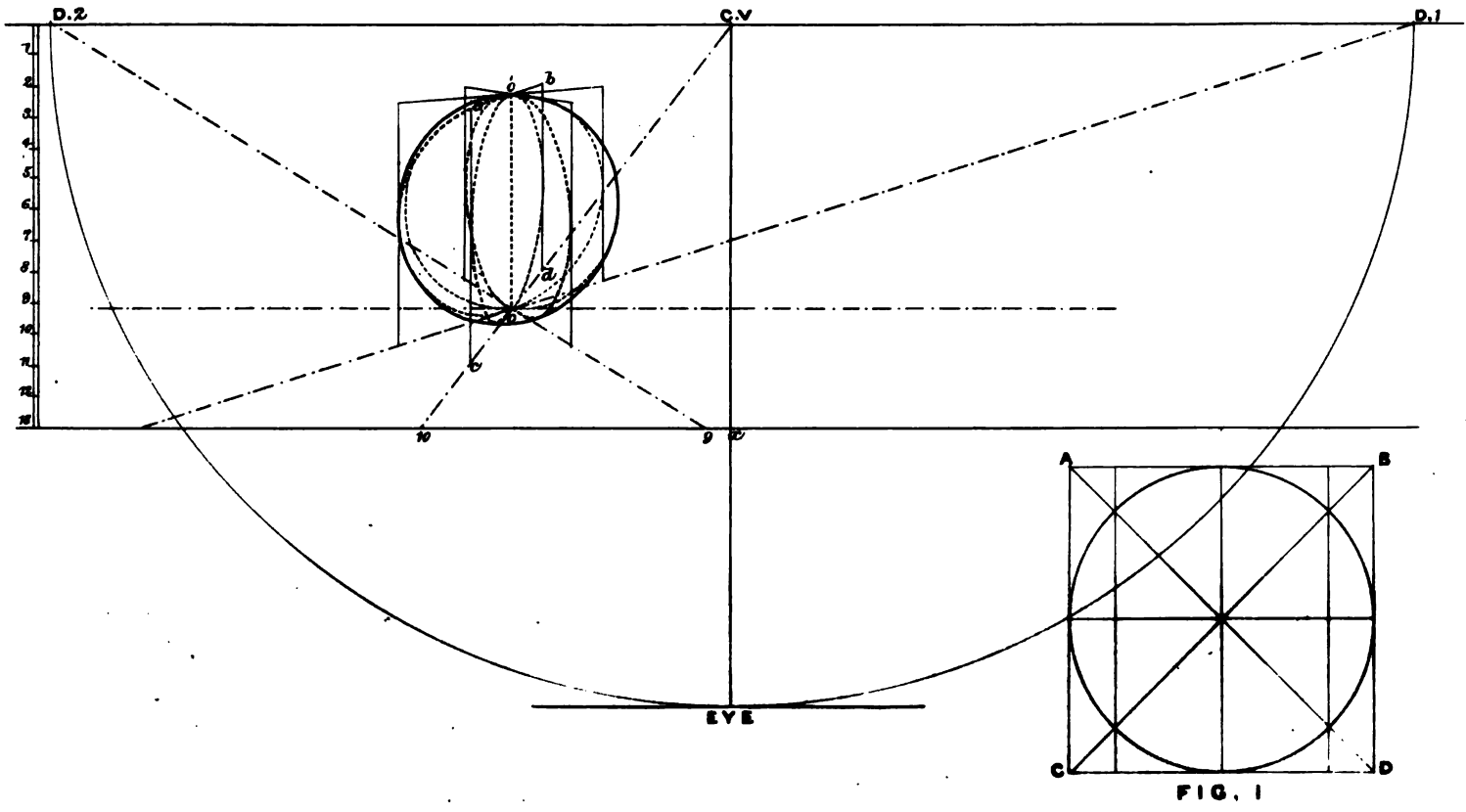


PLATE 52.

Distance of eye in front of the picture-plane, 22'.

Height of eye above the ground-plane, 13'.

Give the perspective representation of a sphere resting upon the ground-plane, having its axis vertical at 10' on the left of the spectator, and 9' beyond the picture-plane.

N.B. The actual form of any section of a sphere is a circle. If a section be taken through the centre of a sphere, a circle of the largest possible size will be obtained, called the "great circle."

The perspective representation of the contour of a sphere can only appear a circle when it has its centre in a direct line with the principal visual ray.

To obtain the perspective representation of the sphere we must employ a series of imaginary vertical section planes passing through the centre of the sphere; in each case the true form of section is the great circle of the sphere.

Fig. 1 represents the true form of section. Enclose it within a square, *A B C D*, and draw the usual lines of construction for obtaining the perspective representation of a circle.

Four vertical section planes are used in this exercise.

First parallel to the picture-plane.

Second perpendicular to the picture-plane.

Third making 45° with the picture-plane to left.

Fourth making 45° with the picture-plane to right.

The number of vertical section planes is not limited, the student may employ any number; the greater the number the more accurately the contour of the sphere will be defined.

The intersection of the four vertical planes of section with the ground-plane are shown by the chain-lines passing through *O*.

Determine the axis, *O O'*, of the sphere at 10' on the left of the spectator, and 9' beyond the picture-plane.

The axis, *O O'*, is the common intersection of the four vertical planes of section.

In each of the four vertical planes the *great circle*, fig. 1, has to be obtained by the method previously described for the umbrella (plate 11).

Having found the great circle in each vertical plane, determine the contour of the sphere by drawing a curved line tangential to these great circles.

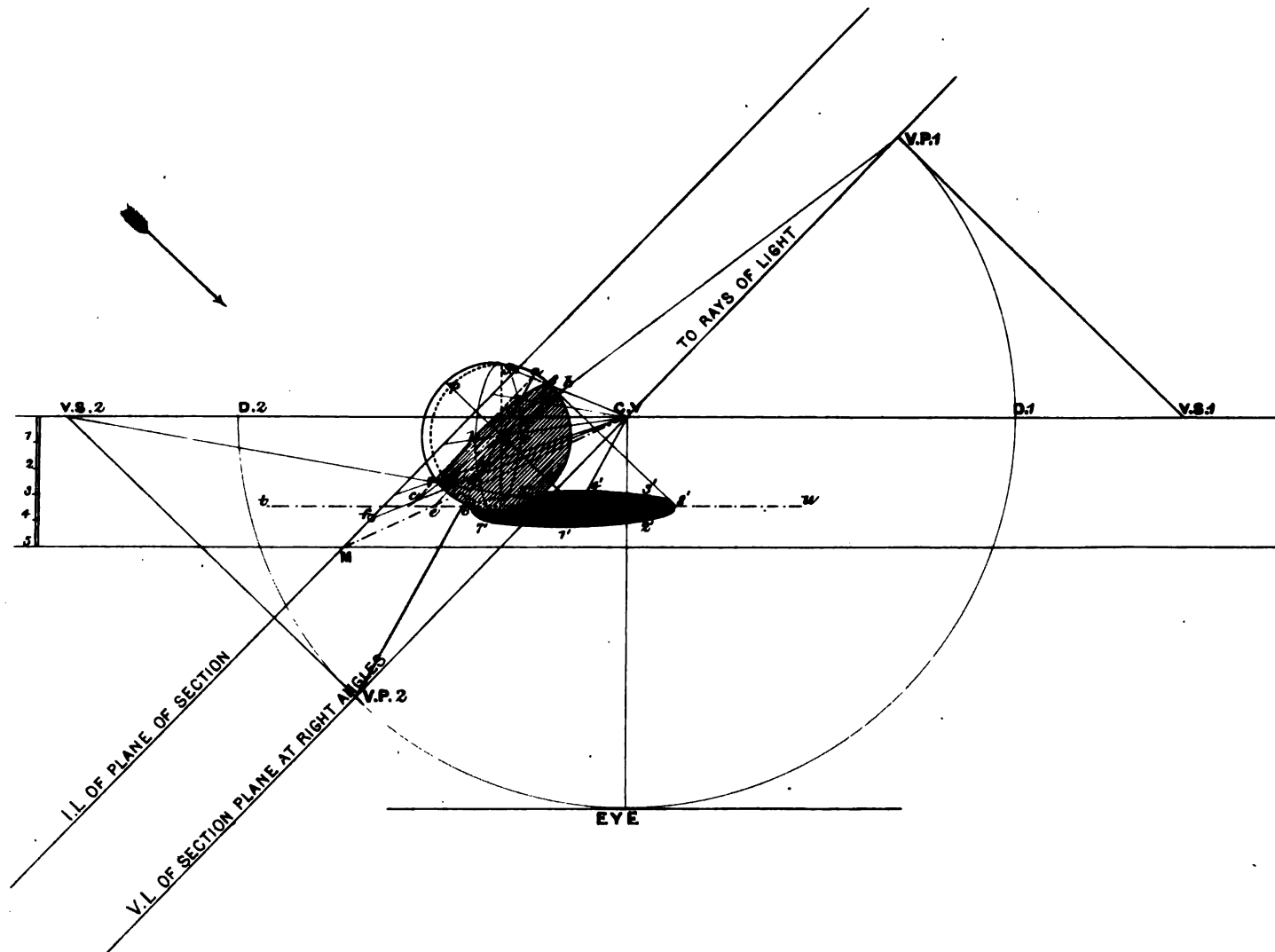


PLATE 53.

Distance of eye in front of the picture-plane, 15'.

Height of eye above the ground-plane, 5'.

Give the perspective representation of a sphere of 8' diameter resting upon the ground-plane, its axis being vertical at 7' on spectator's left and 7' beyond the picture-plane.

Required the cast shadow of the sphere upon the ground-plane when the sun is on the left of the spectator, in the plane of the picture, the rays of light make 45° with the ground-plane.

Since the sun lies in the plane of the picture its rays do not vanish but are represented by parallel lines. If several of these rays be imagined to surround the sphere on all sides, a cylindrical envelope would result, and the shadow of the sphere upon any plane is the imaginary section of this cylindrical envelope by such plane.

The cylindrical envelope is tangential to the sphere in the great circle which lies in a vertical plane passing through the centre of the sphere at right angles to the rays of light.

The sun being in the plane of the picture, it is necessary to obtain a section through the centre of the sphere made by a plane parallel to the picture.

The intersection of this vertical section plane with the ground-plane is shown by the chain-line $t\ u$, drawn through the lower end of the axis, and the section of the sphere made by this plane is the dotted circle which stands vertically upon the chain-line $t\ u$.

Draw a ray of light, $p\ s$, through the centre (o) of the sphere, lying in the vertical section-plane above mentioned.

Produce $p\ s$ to meet the ground-plane on the chain-line $t\ u$, at o' .

Point o' is the centre of the cast shadow of the sphere upon the ground-plane.

Draw the diameter, 68 , of great circle (dotted circle above referred to) perpendicular to the ray of light $p\ s$, and lying in the same vertical plane of section.

Points 6 and 8 are two of the tangential points (*highest and lowest*) of the imaginary cylindrical envelope.

Produce the line 68 to meet the ground-plane on chain-line $t\ u$, at e .

We must now determine the $V\ L$ of a plane passing through the centre of the sphere perpendicular to the rays of light.

This plane must necessarily be perpendicular to the picture making an angle of 45° with the ground-plane; therefore, draw its $V\ L$, through $C\ V$, at right angles with the rays of light.

It contains the line 68, and intersects the ground-plane in the chain-line **M**, e, **C V**; its intersection with vertical plane passing through the sun and containing the axis of sphere, is the line e 68; and its **I L** with the plane of the picture is drawn through **M** parallel to its **V L**.

The section of the sphere made by the plane *perpendicular to the rays of light* is the line of contact of the imaginary cylindrical envelope, it is also the line which separates the light from shadow on the surface of the sphere.

We next determine the curved line of contact. Bring forward points 6 and 8, by **C V**, to meet **I L** of plane of section at **f** and **g**, which equals the actual length of the diameter, 68, of the required section.

Through the centre of the sphere (o) draw lines to **V P 1** and **V P 2**, produce these lines to meet **g C V**, **f C V**, at points **a**, **b**, **c**, **d**.

Join **a c**, **b d**, which will give the perspective representation of the square enclosing the required section of the sphere.

Find the points **2**, **3**, **5**, **7** upon the diagonals of the square, and draw an ellipse through these points.

The shadow of this ellipse will be the cast shadow of the sphere.

Draw rays of light from 8 and 6 to meet the ground-plane on chain-line **t u**, at points 8' and 6'.

These rays are not shown in the plate to prevent a multiplicity of lines.

Now draw a line from **O'**, the centre of cast shadow upon the ground-plane, to **C V**, and to meet this line at points 1' and 4'. draw the rays of light from points 1 and 4 in the section.

Through **V P 1**, **V P 2**, draw lines parallel to the rays of light to meet **V L** of ground at **V S 1**, **V S 2**.

These points are the vanishing points of the shadows of the diagonals of the square enclosing the section; therefore through **O'** draw a line to **V S 1**, and to meet this line at points 3', 7' draw the rays of light from 3 and 7 in the section.

Finally, join **O'** to **V S 2**, to meet this line at points 2', 5' draw rays of light from 2, 5 in the section.

An ellipse drawn through the points 1', 2', 3', 4', 5', 6', 7', 8' upon the ground-plane is the cast shadow of the sphere.

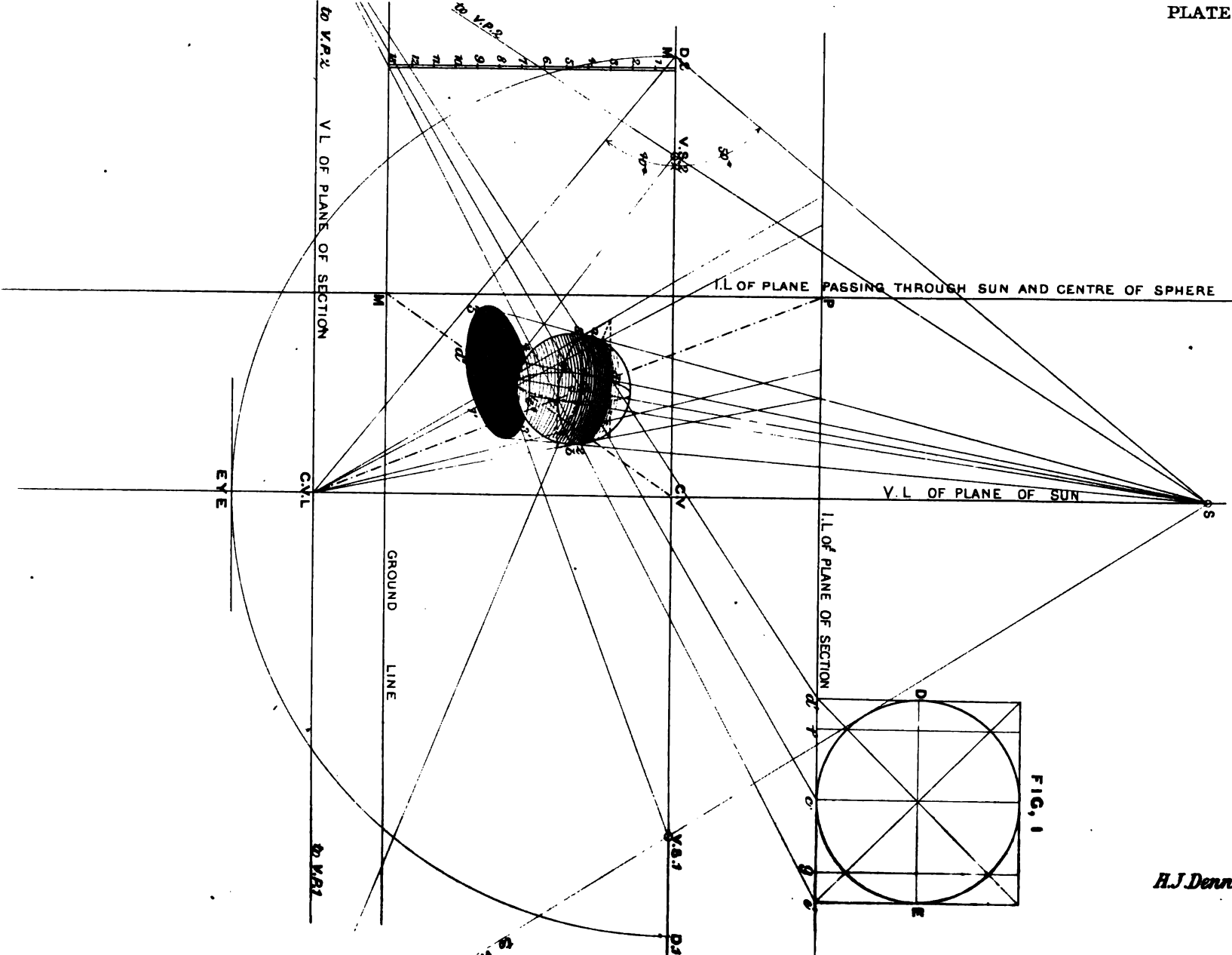


PLATE 54.

Distance of eye in front of the picture-plane, 20'.

Height of eye above the ground-plane, 13'.

A sphere, 10' diameter, rests upon the ground-plane, its axis is vertical at 9' on the left of the spectator, and 18' beyond the picture-plane.

Required the cast shadow of the sphere upon the ground-plane when the sun lies in a vertical plane perpendicular to, and *behind* the picture-plane, the rays of light make an angle of 50° with the ground-plane.

N.B. In this case the rays of light converge to a point. If we imagine the sphere to be encompassed by the rays of light, a conical envelope would result therefrom, the sun would be its apex, and the line of contact of the envelope with the surface of the sphere would be the line of separation of light and shadow.

The line of contact of the conical envelope with sphere, is that portion of the sphere which casts the shadow upon any plane.

The position of the line of contact is always found by drawing the external rays of conical envelope, then through their points of

contact with the surface of the sphere draw an imaginary section of both figures perpendicular to the axis of the conical envelope.

Having found the perspective representation of the sphere and the position of the sun ($\odot S$) behind the picture plane, we must necessarily obtain the IL of a vertical plane passing through the sun and containing the axis of the sphere.

Trace out the end of the axis upon the ground-plane by CV to meet the ground-line at M . Through M draw the IL of plane above referred to.

The section of the sphere made by this vertical plane is represented by the vertical ellipse having O for its centre.

Draw rays of light from $\odot S$ tangential to the contour of the sphere at a and b .

Join a, b , this line cuts the ray of light drawn from $\odot S$ to O (centre of sphere) at point c .

Now draw the VL and IL of a plane containing a, b , and c , perpendicular to the central ray of the conical envelope; this plane

gives the imaginary section of the envelope and sphere, the darkest part of the shadow on the surface of the sphere.

At **M** (**D 2**) make a right angle with the altitude of the sun, or 40° with horizon; produce this line to meet the **VL** of plane of sun at **CVL**. Through **CVL** draw the **VL** of plane of section, trace out **e** by **CVL** to meet the picture at **P**; through **P** draw **IL** of plane of section parallel to ground-line.

The chain-line **P, CVL**, is the intersection of the plane of section with the plane which contains the axis of conical envelope and sphere; therefore **de** must be a diameter of the required section.

The actual length of **de** should be now ascertained.

Bring forward **d** and **e** by **VP 2** (**D 2** on **VL** of plane of section) to meet **IL** of plane of section at **d', e'**. Construct a square upon **d' e'**, and within it inscribe a circle, fig. 1, which represents the true form of section of the conical envelope and sphere made by the plane previously referred to.

Draw the front and back edges of the square enclosing the section through points **d, e**; the perspective lengths of these lines are determined by setting off the distances **ed', ee'**, fig. 1, on either side of **P**, and joining these points to **CVL**.

Draw the diagonals of the square to **VP 1** and **VP 2**. Set off **ef, eg**, fig. 1, on either side of **P**, and join the points to **CVL**, these lines cut the diagonals at points **1, 3, 4, 6**, through which the curved line of section is drawn.

We have now to find the cast shadow of this section upon the ground.

Draw a ray of light through the centre of the sphere to meet the ground-plane at **O'**, on the chain-line **M, CV**.

The points **d** and **e** in the curved line of section lie in the vertical

plane which contains the axis of the sphere and the sun; therefore, if we draw rays of light from **d** and **e**, these rays must necessarily meet the ground on the chain-line **M, CV**, at **d', e'**.

The points **1** and **4** lie on the diagonal of the square which has **VP 2** for its vanishing point; therefore join **VP 2** to **OS**, giving **VS 2** upon the horizon (**VL** of shadow-plane).

Join **O'** to **VS 2**, then draw rays of light through points **1** and **4** to meet the former line at points **1'** and **4'**.

Join **O'** to **VS 1**, and upon this line points **5'** and **3'** are obtained.

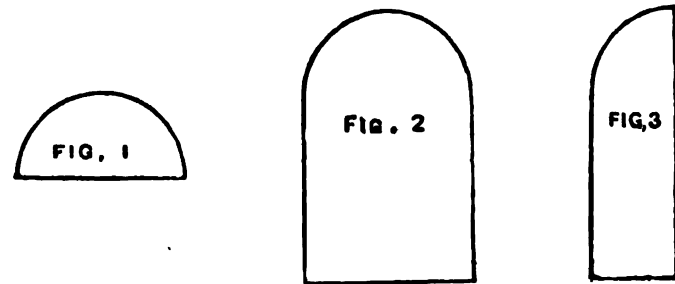
Finally, draw the curved line of cast shadow through the points thus obtained.

TO FIND THE SHADOW OF A NICHE.

A niche is composed of a concave semi-cylinder, and a portion of a concave sphere. It is a recess within the thickness of a wall for a statue or bust.

The shadow of a niche is obtained by the method already described for cylinders and spheres.

Figures 1, 2, and 3 represent the plan and two elevations of a niche.

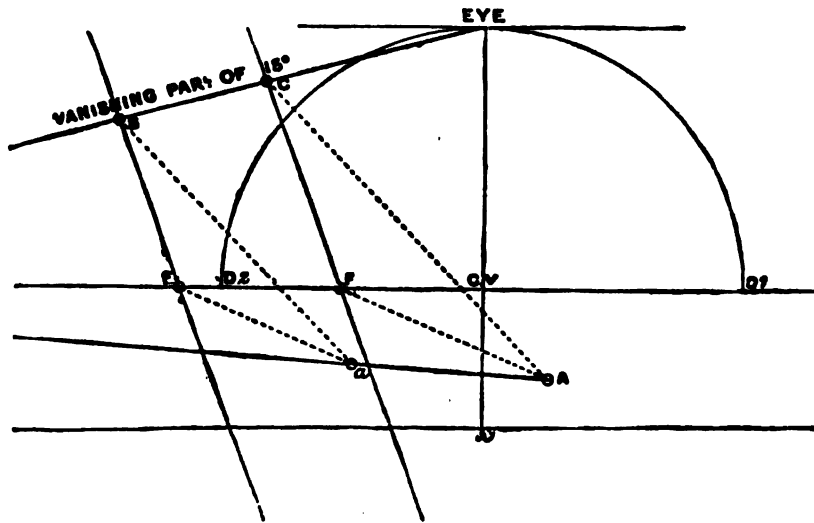


A series of sections of the niche made by planes passing through the sun should be found. If these planes cut the cylindrical portion of the niche only, the true form of section in each case is a semi-ellipse.

If the sun be behind, or in front of the picture, and the section-planes passing through it cut the spherical head as well as the cylindrical part of the niche, the true form of section is partly circular and partly elliptical.

TO DRAW A LINE THROUGH ANY GIVEN POINT WHEN ITS VANISHING POINT IS INACCESSIBLE.

Let A be the given point lying on the ground through which the



required line has to be drawn at an angle of 15° with the picture-plane towards spectator's left.

Draw the vanishing parallel of 15° towards left, it will be found to meet the horizon somewhere outside the surface of the paper, consequently the vanishing point of the line is inaccessible; there is, however, a very simple method by which we can draw a line from the given point A, which would, if produced far enough, meet in its vanishing point.

Draw any two *parallel* lines C F, B E, then join the given point A to C and F so as to form a triangle.

Upon the line B E construct a *similar* triangle by drawing B a parallel to C A, and E a parallel to F A.

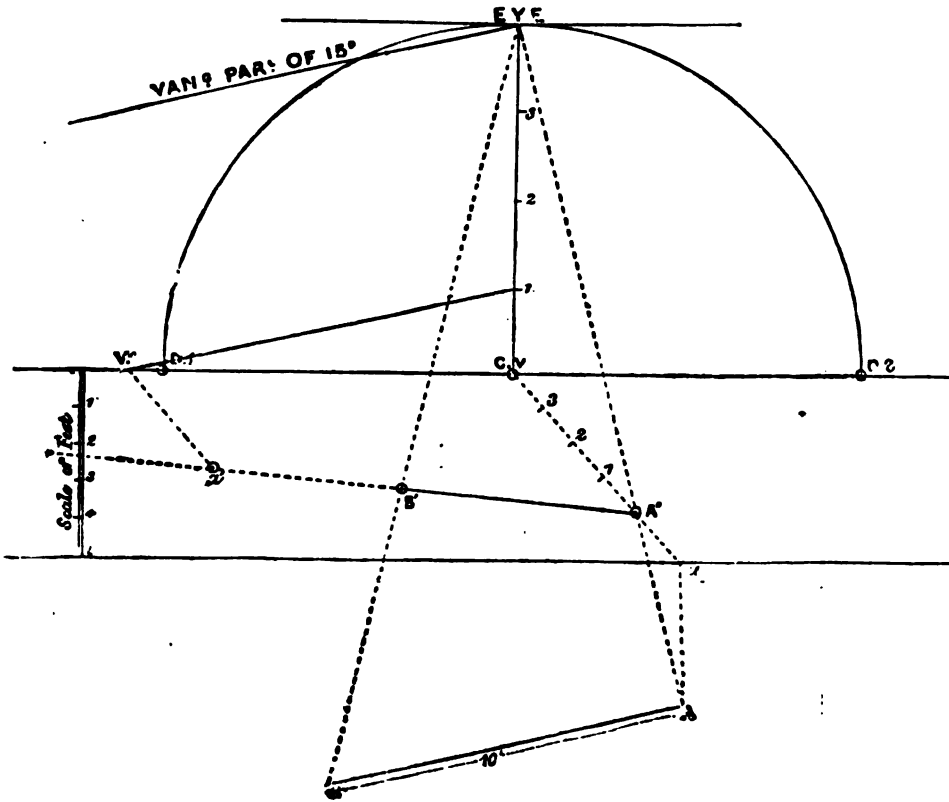
Finally, join A a which is the line required.

When the line is required of a certain perspective length, the method given below will be found useful.

Let A' be the given point through which it is required to draw a line 10' long at an angle of 15° with the picture-plane towards the spectator's left. (See next page.)

Divide the *principal visual ray* into any number of equal parts (say 4). Through point 1 draw a line to meet the horizon at V' parallel to the vanishing parallel of the required line. Join A' to C V and divide the line into *four* equal parts, because the principal visual ray is divided into this proportion.

Draw a' V' parallel to A' C V, and equal to *three fourths* the length of this line. Join A' a', which, if produced, would meet the horizon at its vanishing point.



Now determine what length point **A'** is beyond the picture in a *perpendicular* direction. Bring forward **A** by **CV** to meet the ground-line at **a**, let fall a perpendicular from **a**, and to meet it at **A**, draw the ray of light from the **EYE** through point **A'**.

The line $a \Delta$ is equal to the actual distance of given point Δ' beyond the picture.

From **A** draw a line **A B**, 10' long, at an angle of 15° with the ground-line, then join **B** to the **EYE**, this ray of light determines the perspective length of the required line at **B'**, upon the line **A' a'** previously obtained.

PROJECTIONS OF SHADOWS BY ANY ARTIFICIAL LIGHT.

The theory of shadows projected by candle, lamp, or gas-light, is precisely that which is given for the projections of shadows by the sun.

The artificial light is always conceived to be situated within the field of vision *behind* the picture-plane.

The position of the artificial light is shown by a simple point, which is determined upon the same principle as any other point.

The rays of light are straight and converge to a point in the centre of the flame of the candle, &c.

To find the shadow of a given line upon any given plane.

GENERAL RULE. Imagine a plane to pass through the centre of the flame, and contain the given line, the intersection of this plane with the plane on which the shadow is required, is the shadow of the given line.

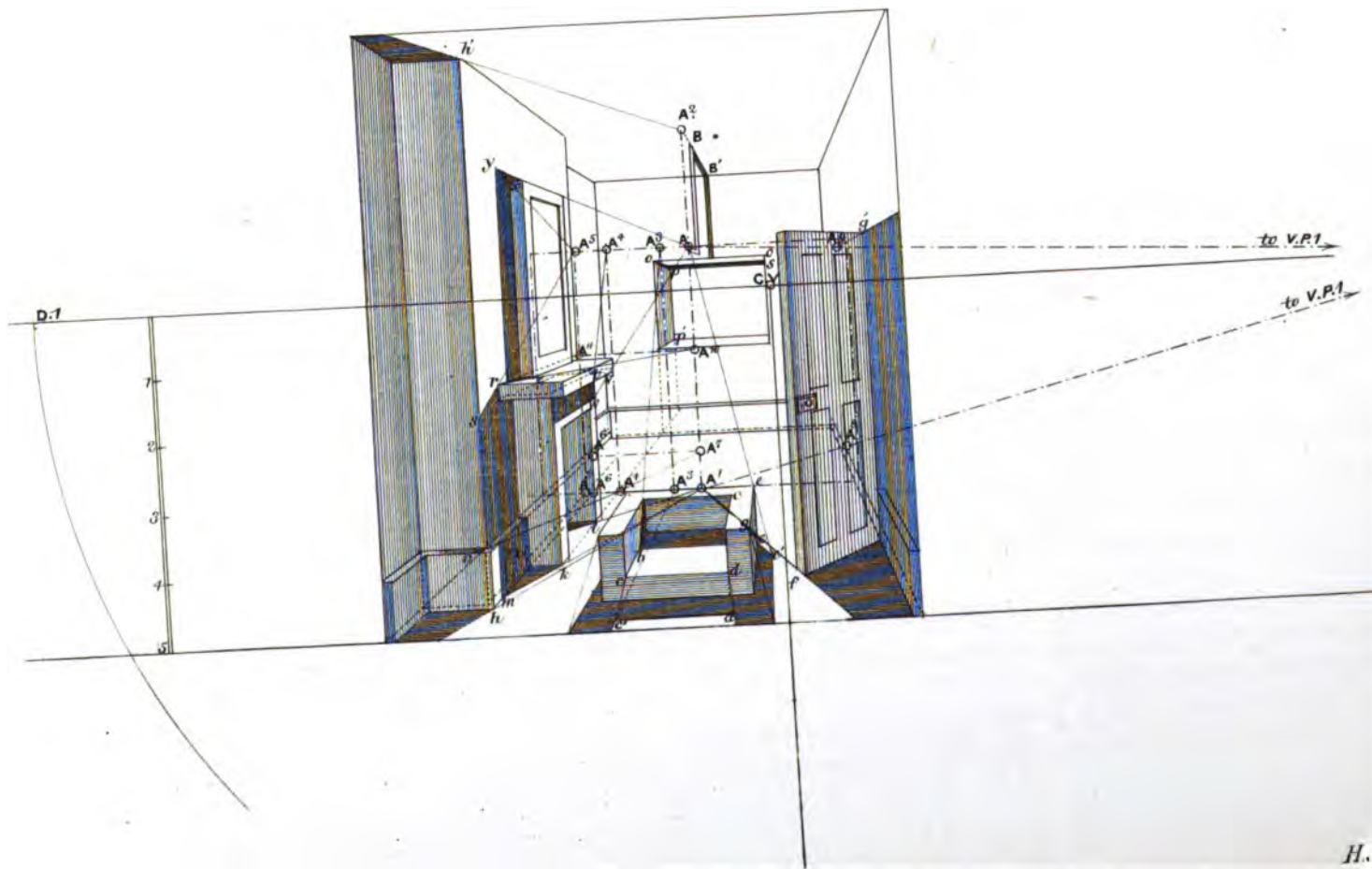


PLATE 55.

Distance of eye in front of the picture-plane, 12'.

Height of eye above the ground-plane, 5'.

Given the perspective representation of a room and the position of a gas-burner projecting from the ceiling, to find the shadows projected by the gas-light.

1st. *Required the shadow of the fire-place and the rectangular mirror which rests upon the mantel-piece.*

By making a careful investigation of what has to be done, the student will not fail to discover that the lines of the fire-place and mirror which cast shadows, are those which are horizontal and vertical, parallel to the plane of the picture; therefore, through \mathbf{A} (the centre of the gas-flame) draw a vertical plane parallel to the picture-plane, and show its intersection with the floor and opposite walls of the room.

Its intersection with the floor is the chain-line $\mathbf{A'A^5}$; and its intersection with the left hand projecting wall containing the fire-place is shown by the chain-line $\mathbf{A^5A^5}$.

Now draw lines through \mathbf{A} parallel to the edges of fire-place which cast shadows, and where these lines meet the planes on which

the shadows are required, we shall have the vanishing points of the shadows.

It is obvious from what has been said above, that point $\mathbf{A^5}$ is the vanishing point of the shadows *upon the projecting wall* of the horizontal edges of the fire-place and mirror which are parallel to the plane of the picture. Point $\mathbf{A'}$ is the vanishing point of the shadows of the vertical edges upon the *floor*.

The shadow of the nearest upright edge of fire-place upon the ground-plane is the line $\mathbf{k m}$, vanishing to $\mathbf{A'}$.

This upright edge also casts a shadow upon the skirting-board in the vertical line $\mathbf{m n}$, a continuation of this shadow is now required upon the upper thickness of skirting-board.

The vanishing point of the shadow $\mathbf{n o}$ is determined by imagining the horizontal plane of upper thickness of skirting-board produced to meet the vertical plane parallel to picture passing through gas-flame. These two planes intersect each other in the chain-line $\mathbf{A^6A^7}$, and a line drawn through \mathbf{A} (gas-flame) parallel to upright edge of fire-place meets the level of the top of skirting-board at $\mathbf{A^7}$, which is the vanishing point of the shadow $\mathbf{n o}$.

Through \mathbf{r} draw $\mathbf{r s}$ to $\mathbf{A^5}$. The line $\mathbf{r s}$ is the shadow of a portion of the upper horizontal edge of mantel-piece on the projecting wall of fire-place.

The mantel-piece projects beyond the face of the fire-place, and must necessarily cast a shadow upon it.

We have now to find the intersection of the plane of face of fire-place with the vertical plane passing through the gas-flame. The chain-line $A^4 A^4$ represents it.

Produce the furthest upright edge of fire-place to meet the under surface of the mantel-piece at point t , then draw $t v$ parallel to the picture-plane. Join t to A^4 , and produce the line upon the face of the fire-place, a ray of light drawn from A through point v will meet the line $t A^4$ produced in point w .

w is the shadow of point v , and since the long edge of mantel-piece is parallel to the face of the fire-place, its shadow must have the same vanishing point, viz., $C V$.

The inner upright edge of fire-place farther from the spectator cast a shadow upon the hearth-stone. Draw the shadow from l to A' .

We now turn our attention to the shadow of the rectangular mirror upon the upper surface of the mantel-piece, and the projecting wall of fire-place.

Imagine the upper surface of the mantel-piece produced to cut the vertical plane parallel to picture passing through gas-flame.

These planes intersect in the chain-line $A^{10} A^{11}$.

A^{10} is the vanishing point of the nearest upright edge of the rectangular mirror upon the mantel-piece.

Through point z draw a line to A^{10} , and where it meets the surface of the projecting wall of fire-place raise a perpendicular, then draw through x a line to A^5 to meet the perpendicular at y .

The upper edge of mirror being parallel to the projecting wall, its shadow is drawn from y to $C V$.

The upright edge of the skirting-board casts a shadow upon the floor to represent which we must join h to A' .

The projecting wall of fire-place will also cast a shadow upon the plane of the ceiling.

Find a point on the ceiling exactly opposite the centre of the gas-flame at A^2 , then draw the shadow from h' to A^2 .

2nd. Required the shadow of the upper surface of recess at back of room, upon the inner wall.

The vertical plane $o p'$ forming the left side of the recess is a continuation of the plane of face of fire-place, consequently the chain-line $A^4 A^4$ represents its intersection with the vertical plane parallel to picture passing through A ; and since A, A^4 is parallel to the line $o o$ which casts the shadow, A^4 must be the vanishing point of the shadow upon the surface $o p'$ of the recess.

Through o draw $o p$ to A^4 , the continuation of the shadow ($p s$) upon the back wall of recess is drawn parallel to $o o$, because the line and shadow plane are parallel.

3rd. Required the shadow of the stem of gas-burner upon the planes of the ceiling and back wall of the room.

Point A^2 is the vanishing point of the shadow upon the ceiling, because the line $A A^2$ is parallel to the line which casts the shadow and meets the shadow plane at A^2 .

Through B draw $B B$ to A^2 , the remainder of the shadow upon

the back wall should be vertical because the line and shadow plane are so situated.

4th. Required the shadow of the four slabs upon the floor, also the shadow of the inner surface of farther slab upon the interior surfaces of the other slabs.

The upright edges of the slabs have their shadows upon the floor, vanishing to A' , as previously explained for the vertical edges of the fire-place.

Join A' to e , and produce the line towards the picture-plane, then draw the ray of light Ae and produce it to meet the floor upon former line at e' . Line ee' is the shadow of ee .

The edge of slab eg is parallel to the floor (shadow plane) therefore join e' to $C V$. The shadow of g is determined by drawing a ray of light from A through g to meet the floor.

Let fall perpendiculars from c and d to meet the floor, and through the points thus obtained draw lines to A' .

Upon the lines last obtained the shadows of c and d are determined by drawing rays of light from A through c , d , to meet the floor.

Join $c' d'$ for the shadow of line cd .

The remainder of the shadow of the slabs upon the floor is so simple that it does not require further explanation.

We have now to find the shadow of ao upon the interior surfaces of the slabs.

Imagine the inner vertical plane of slab (ac) produced to meet the vertical plane parallel to picture passing through the gas-flame at $A^s A^s$; then, because AA^s is parallel to the line ao and meets the plane of inner surface of slab (ac) at A^s , the shadow of ao upon this vertical plane, has its vanishing point at A^s . Through a draw ab to A^s , and because ao is parallel to the upper surface of the horizontal slab, the remainder of its shadow is drawn from b parallel to ao .

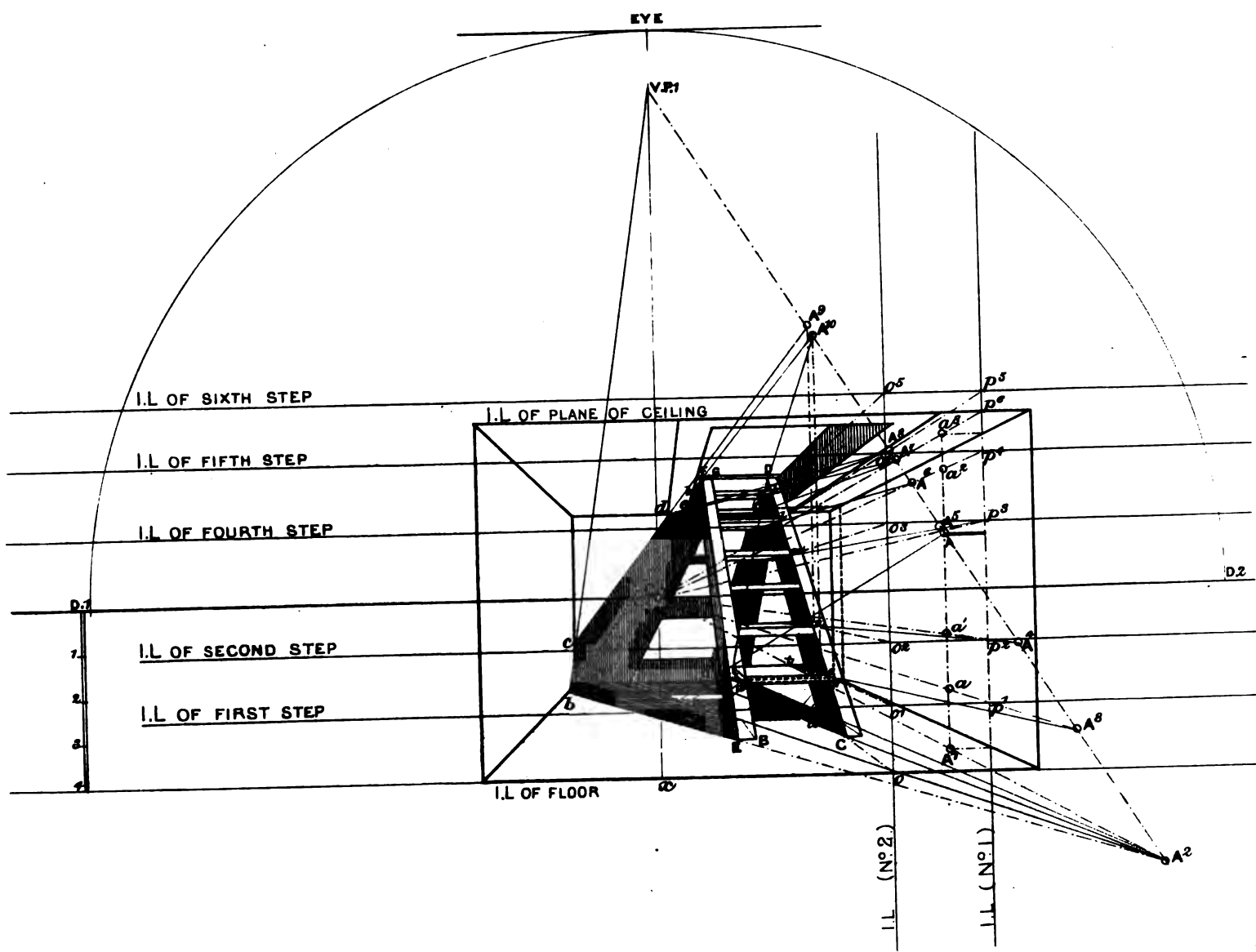
5th. Required the shadow of the door, which is opened at a given angle, upon the floor and wall of room.

The upright edge of door has the vanishing point of its shadow upon the floor at A' . Draw a right line from f to A' .

In order to find the vanishing point of the shadow of the upper edge of door upon the surface of the wall, we must imagine a vertical plane passing through the gas-flame and the vanishing point of the line whose shadow is required, and where this plane intersects the plane of the wall we shall have the vanishing point of the shadow.

The above mentioned plane intersects the floor in the chain-line drawn from A' to $VP 1$; it cuts an imaginary horizontal plane containing the gas-flame in the chain-line drawn from A to $VP 1$; it also cuts the plane of the wall in the chain-line $A^s A^s$.

Point A^s is the vanishing point of the shadow. Join g' to A^s .



H.J. Dennis

PLATE 56.

Distance of eye in front of the picture-plane, 12'.

Height of eye above the ground-plane, 4'.

Given the perspective representation of a store-room, in the ceiling of which there is an entrance to a loft. Required the shadow of the steps upon the surfaces of the floor and walls of the store-room, also upon the vertical surface of the aperture in the ceiling. The position of the gas-burner being given at point $\odot A$.

Obtain $VP1$, the vanishing point of the inclined sides of steps, also obtain the intersection with the floor of the store-room, *or that plane produced*, of a vertical plane passing through $VP1$ and $\odot A$, the centre of the gas-flame.

The intersection of the above planes is represented by the chain-line A^2, CV .

The shadows of the inclined edges of the sides of the steps are determined by a series of imaginary planes passing through $\odot A$ containing the inclined edges; the intersections of these planes with those of the floor and walls of the store-room give the shadows.

It is my intention to explain the method of finding the shadow of one of the inclined edges (say EF), the other edges being parallel to EF , their shadows will have the same vanishing points.

The plane which passes through $\odot A$ and the inclined edge EF ,

cuts the floor of the store-room in the chain-line A^2Eb . The line Eb is therefore the shadow of a portion of the line EF upon the floor.

The line EF and the left hand wall of the store-room are parallel, consequently the shadow upon this plane must vanish where the line vanishes, viz. $VP1$.

Draw bc to $VP1$.

We must now determine the intersection of the back wall of the store-room, *produced upwards*, with the vertical plane passing through $\odot A$ and $VP1$.

Upon careful observation the student will perceive that the two planes cut each other *upon the floor* at point 1, and since the two planes are vertical, it is obvious their intersection is the vertical chain-line drawn from point 1 to meet the chain-line $A^2, VP1$, at A^2 .

Point A^2 is the vanishing point of the shadow of EF upon the back wall of the room.

Draw ed to A^2 .

As the shadow of the line EF does not terminate upon the back wall of the room, it is very evident the intersection of the plane of the ceiling with the vertical plane passing through $\odot A$ and $VP1$ must be determined.

The latter plane (*vertical plane passing through $\odot A, VP1$*) has its intersection with the picture-plane at IL (No. 1) which cuts the IL of the plane of the ceiling at point p^2 . The two planes are *per-*

pendicular to the picture-plane, therefore their intersection must also be perpendicular to the picture-plane and vanish to $C V$; chain-line $p^6 C V$ represents it.

The intersection of these planes (chain-line $p^6 C V$) gives point A^7 on the chain-line $A^2, V P 1$.

Point A^7 is the vanishing point of the continuation of the shadow of $E F$ upon the ceiling.

Draw $d e$ to A^7 .

We have finally to obtain the intersection of the vertical surface at the back of the aperture in the ceiling, with the vertical plane passing through $\odot A$ and $V P 1$.

Imagine the above mentioned vertical surface of the aperture produced to meet the floor of the store-room in the dotted line which crosses the chain-line $V P 1, A^2$, at point 2. The vertical chain-line $2, A^{10}$, is the intersection of the two planes.

Point A^{10} is the vanishing point of the shadow of the remaining portion of the line $E F$.

Draw $F k$ to A^{10} .

TO FIND THE SHADOW OF THE INCLINED EDGE, $C D$, UPON THE HORIZONTAL SURFACES OF THE STEPS.

Obtain $I L$ (No. 2) the intersecting line of the vertical plane, containing $C D$, then trace out the intersections of the *upper* surfaces of the *first* and *second* steps, and the *lower* surfaces of the *fourth*, *fifth* and *sixth* steps with this vertical plane.

The chain-lines $C V O^1, C V O^2, C V O^3, C V O^4, C V O^5$, represent the intersections.

Through the points O^1, O^2, O^3, O^4, O^5 , draw the $I L$'s of the steps.

Now obtain the intersections of the horizontal planes of the steps with the vertical plane passing through $\odot A$ and $V P 1$.

These intersections are represented by the chain-lines $C V p^1, C V p^2, C V p^3, C V p^4, C V p^5$, and the points a, a^1, a^2, a^3 , &c., are points lying in the several planes immediately *under* and *above* the centre of the gas-flame. These points are called "*seats*" of the *luminary*.

If the chain-lines $C N p^1, C V p^2$, &c., be produced to meet the chain-line $V P 1, A^2$, we shall have the vanishing points of the required shadows at A^3, A^4, A^5, A^6, A^7 .

The shadow of $C D$ upon the first step is the line $s t$, drawn to A^3 , upon the second step it is shown by the line drawn from n to A^4 , a continuation of the shadow upon the fourth step is drawn from m to A^5 , upon the fifth step it is again represented by the line drawn from l to A^6 ; and upon the sixth step the line $h i$ is drawn to A^7 .

The shadows of the steps upon the floor and back wall of store-room are so easily obtained that the *first* need only be explained.

The points $s w$ lie upon $C D$, therefore if we draw rays of light from $s w$ to meet the shadow of $C D$ upon the floor we shall have the shadows of the points at s and w ; now find the shadows of points $y x$ on the shadow of inclined line $B G$.

Join shadow points $w x$ on the floor, which will give the shadow of lower edge of step; the remaining part of the shadow of the first step is found by drawing rays of light to meet the floor, through the upper extremities of the back edge of the step.

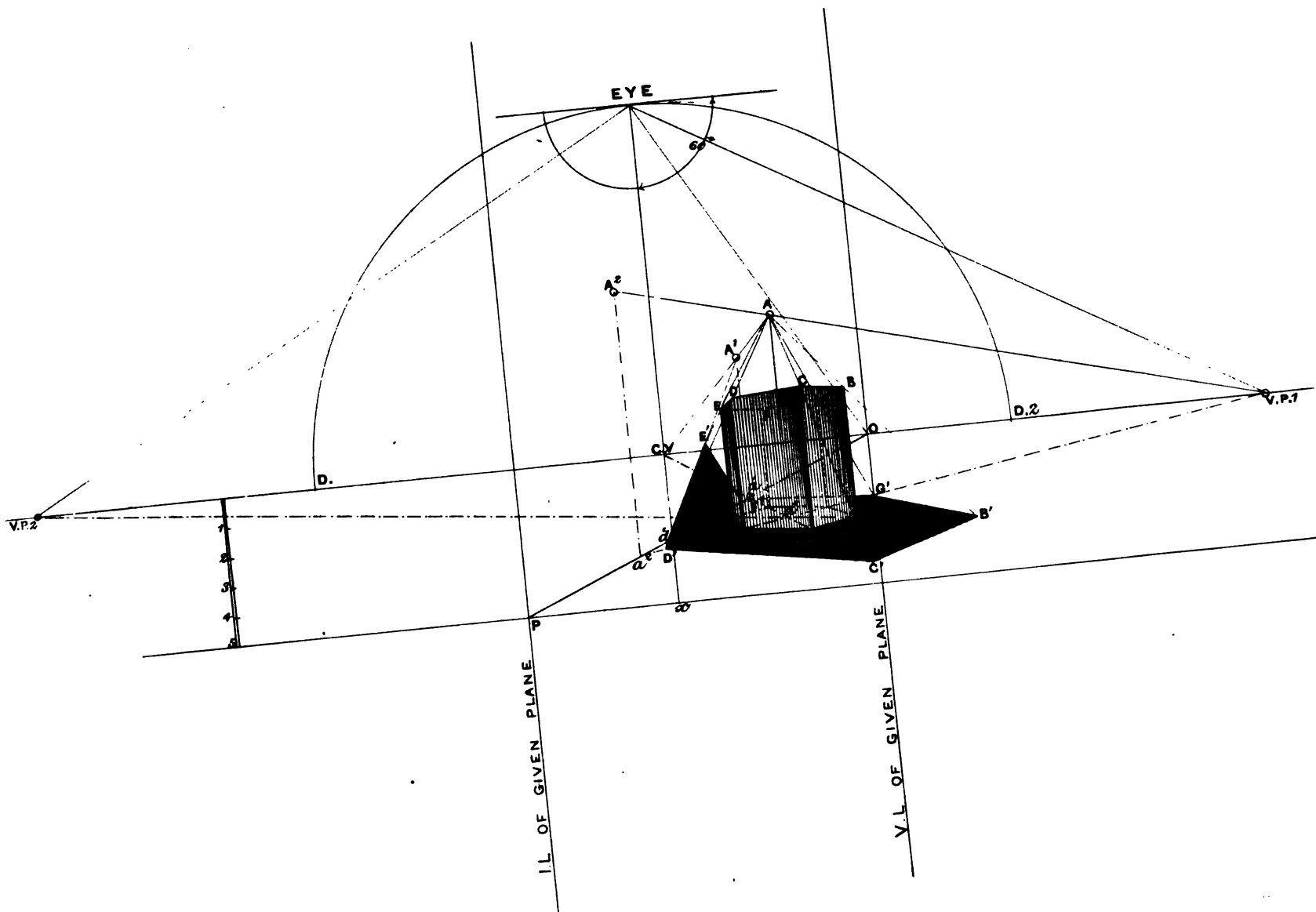


PLATE 57.

Distance of eye in front of the picture-plane, 12'.

Height of eye above the ground-plane, 5'.

I. A hexagonal prism rests upon the ground-plane on its base, the nearest corner of which is at S' on the right of spectator, and S' beyond the picture-plane. Height of prism 8'. Side of base 4'.

II. A vertical plane intersects the picture at $5'$ on left of $C V$, and recedes from it at an angle of 60° towards spectator's right.

III. A candle rests upon the top of the hexagonal prism, the position of the flame being shown at point $\odot A$. Required the shadow of the prism upon the ground and vertical plane.

Find a point a'' upon the ground-plane immediately below the candle-flame by drawing $\odot A a''$ vertically.

Point a is the "seat" of the flame A , on the upper end of the prism, while a'' is its "seat" upon the ground-plane.

We must now imagine a series of vertical planes passing through $\odot A$ and containing the opposite vertical edges of the prism; these vertical planes have their intersections with the ground-plane in the chain-lines drawn through a'' .

We will imagine the first vertical plane to pass through $\odot A$ and the opposite upright edges of the prism $D G$. This vertical plane has its intersection with the ground-plane in the chain-line drawn through a'' to $V P 1$; upon this chain-line the shadows $D' G'$ will be determined by drawing the rays of light from $\odot A$ through the upper corners of the prism $D G$.

Imagine a second vertical plane to pass through $\odot A$ and the opposite edges of the prism $C F$. This plane is *perpendicular* to the picture plane, therefore its intersection with the ground-plane is the chain-line drawn through a'' to $C V$. Draw the rays of light $\odot A C$ $\odot A F$, to meet the chain-line at C', F' .

We next employ a third imaginary vertical plane passing through $\odot A$ and the opposite vertical edges of the prism B, E . The chain-line drawn through a'' to $V P 2$ is its intersection with the ground-plane, its intersection with the vertical shadow plane is determined by drawing a vertical chain-line from E'' ; then, if a ray of light be drawn from $\odot A$ through E we shall have its shadow upon the *vertical shadow plane* at E' . A ray of light drawn from $\odot A$ through upper corner B of the prism will determine its shadow B' upon the ground-plane.

Join $D' C'$, $C' B'$, $B' G'$, $G' F'$.

The upper edges $D E$, $F E$, of the prism cast shadows upon the ground-plane as well as upon the vertical shadow plane.

It is necessary to observe that the line $D E$ is parallel to the ground-plane and vanishes to $C V$, its shadow ($D' d'$) upon that plane must therefore have the same vanishing point.

Join $d' E'$ for the shadow of $D E$ upon the vertical shadow plane, or it may be obtained thus:—

Imagine a vertical plane to pass through $\odot A$ parallel to the vertical face of the prism ($D E$) which is perpendicular to the picture-plane.

The intersection of this imaginary vertical plane with the ground

is the chain-line $a' C V$, which cuts the intersection of the vertical shadow plane with the ground (chain-line $O P$) at a' .

The chain-line $a' A'$ is the intersection of the vertical plane passing through $\odot A$ perpendicular to the picture-plane, with the vertical shadow plane; consequently, point A' must be the vanishing point of the shadow of $D E$ upon that shadow plane.

The line $F E$ is parallel to the ground, therefore draw its shadow upon that plane to $V P 1$; then, if we imagine a vertical plane passing through $\odot A$ parallel to the vertical plane $F E$, its intersection ($A^2 a^2$) with the vertical shadow plane will give point A^2 as the vanishing point of that portion of the shadow of $F E$ which is cast upon the vertical shadow plane.

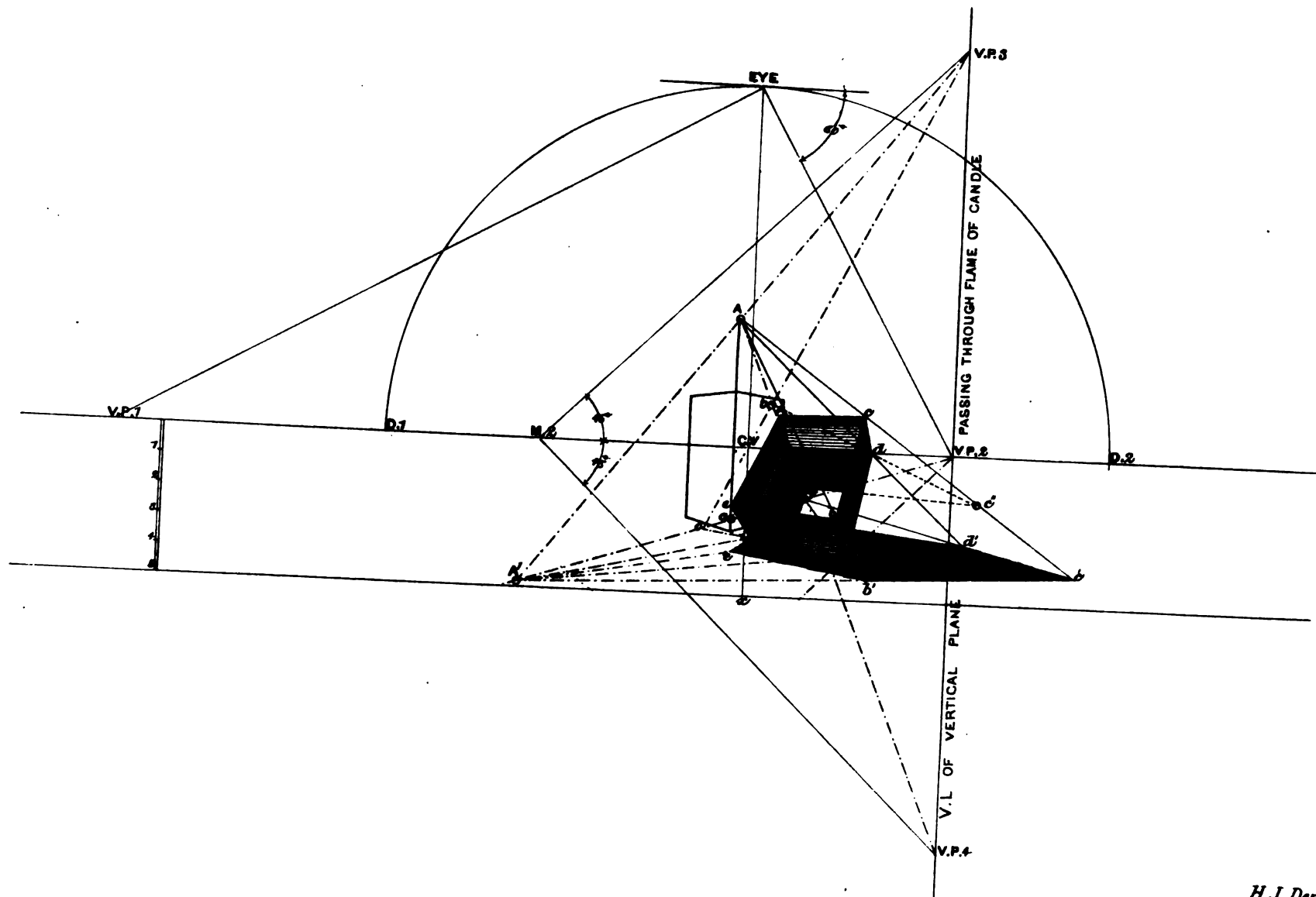


PLATE 58.

Distance of eye in front of the picture-plane, 12'.

Height of eye above the ground-plane, 5'.

Given the perspective representation of a cardboard box, without a lid, its edge upon the ground-plane makes an angle of 30° with the picture-plane towards the left, and the under surface is inclined to the ground-plane at an angle of 45° .

Given also the representation of a square prism whose rectangular faces make equal angles with the picture-plane; upon its upper end rests a candle, the flame of which is represented by the point $\odot A$.

Required the shadow of the box upon the ground-plane, also the shadows of the upper edges upon the lower interior surface of the box.

First find the shadows of the edges ie , eb , bc , of the box upon the ground-plane.

Imagine a vertical plane parallel to the sides of the box $iefg$, $bcpd$, passing through the centre of the candle-flame ($\odot A$) and cutting the ground in the chain-line $A' a VP 2$.

Now join $VP 4$ and $VP 3$ to $\odot A$ and produce the lines to meet the ground-plane on the chain-line $A' a VP 2$ at points A^1 and A^2 .

A^1 is the vanishing point of the shadows of the upper edges bc , ef , of the box upon the ground-plane.

A^2 is the vanishing point of the shadows of the edges ei , bp , cd , fg , upon the ground plane.

Join i to A^2 , and to meet this line draw the ray of light $\odot A e$ to meet the ground at e' . Join p to A^2 and through b draw a ray of light to meet the ground-plane at b' . If the points $e' b'$ be joined the shadow of eb will be determined, which, if accurately drawn, should vanish to $VP 1$, because eb is parallel to the plane of shadow.

A plane passing through $\odot A$ and the upper edge bc of the box cuts the ground in the chain-line $A' b' c'$, the shadow of bc is therefore determined upon this chain-line by drawing the ray of light $\odot A c$ to meet it at c' .

Join c' to A^1 , then draw the ray of light $\odot A d$ to meet it at d' . The line $c' d'$ is the shadow of the short edge, cd , of the box.

Only a part of the shadow of the upper edge, dg , will be seen, and since the line dg is parallel to the shadow plane, we simply join d' to $VP 1$.

N.B. Point d' could be determined another way, thus:—

Imagine a plane to pass through $\odot A$ and the line pd of the box,

this plane would cut the ground in the chain-line drawn from A' through p and the shadow of d must necessarily be on this chain-line. The ray of light drawn through d determines the length of the shadow.

The shadows of the upper edges of the box have now to be found upon the interior lower surface.

Produce the lower edge of the box, upon the ground, to meet the chain-line $A a V P 2$ at O , then join O to $V P 3$ by another chain-line. The latter line represents the intersection of the plane of the bottom of the box with the vertical plane which passes through the flame of the candle parallel to the sides of the box.

The chain-line $\odot A, V P 4$, meets the chain-line $O V P 3$, at O' , which is the vanishing point of the shadow of the edge $f g$ upon the interior of the box.

Join g to O' and produce the line, then draw a ray of light through f to meet at f' , which is the shadow of the upper corner f .

N.B. The line $o' g f'$ is really the intersection of a plane passing through $\odot A$ and the line $g f$ with the interior surface of the box.

A plane passing through $\odot A$ and the upper edge $e f$ of the box intersects the ground in the chain-line $A' e e''$, it also cuts the plane of the lower surface of the box in the chain-line $e'' V P 3$, therefore the shadow of $e f$ is $e'' f'$.

Finally, we imagine a plane to pass through $\odot A$ and the edge of the box $c d$, this plane cuts the plane of the lower surface of the box in the line $O' d c''$ and gives $d c''$ as the shadow of $d c$ upon the produced plane of box.

Or, since $f c$ is parallel to the plane of the bottom of the box, its shadow must evidently vanish to $V P 1$, then if f' be joined to $V P 1$ and produced to meet the ray of light $\odot A c$ it will give point C'' where it was previously obtained, and consequently prove the construction.

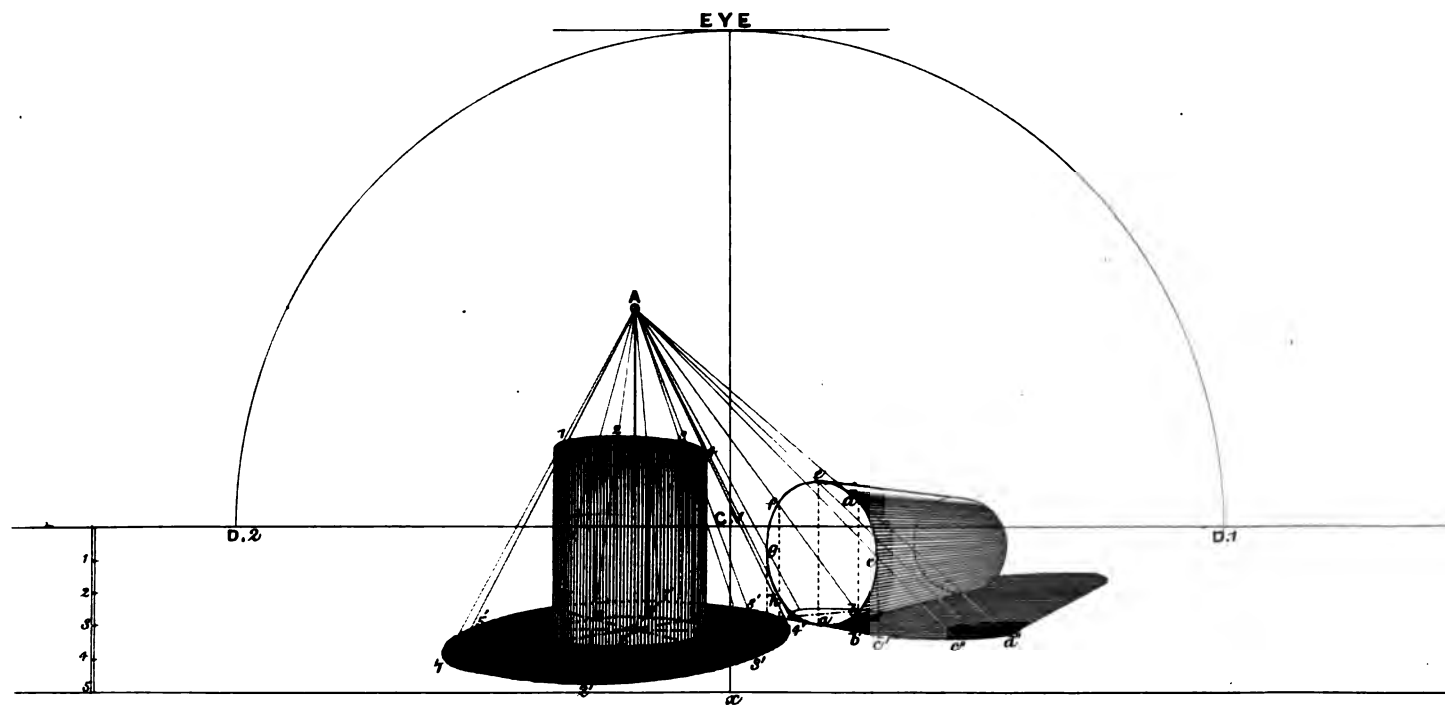


PLATE 59.

Distance of eye in front of the picture-plane, 15'.

Height of eye above the ground-plane, 5'.

Given the perspective representations of two cylinders, one stands vertically upon the ground plane, the other lies upon the ground on its side at an angle of 45° with the picture-plane towards the right.

Required the shadows of the cylinders upon the ground-plane when the position of the flame of the candle is at $\odot A$.

Imagine a series of sections of the vertical cylinder made by vertical planes passing through $\odot A$ and the axis.

Four vertical sections are shown in the plate, viz., parallel and perpendicular to the picture and inclined at 45° to right and left.

The first vertical section of the cylinder is shown by the rectangle 54, 54. Produce the intersection with the ground of this vertical section plane (chain-line 5 A' 4) to right and left, then, if we imagine the rays of light to pass through upper points 5 and 4 and lie in the section plane, their intersections with the ground-plane (points 5', 4') will give the shadows of the points through which they pass.

The second vertical section, perpendicular to the picture, is represented by the rectangle 27, 27. Produce the chain-line 2 A' 7 towards the spectator, and beyond the cylinder, then draw the rays of light through the upper points 2, 7, and produce them to meet the ground-plane at 2' and 7'.

The third vertical section is shown by the rectangle 18, 18. Produce the chain-line 1 A' 8, then draw the rays of light A 1, A 8, to meet it at 1' and 8'.

Points 3' and 6' are found in the fourth section plane upon the same principle as 5', 4'; 2', 7'.

Join 1', 2', 3', 4', 5', 6', 7', 8' for the contour of the shadow of the upper end of the cylinder upon the ground-plane.

TO FIND THE SHADOW OF THE HORIZONTAL CYLINDER.

Find A', the seat of $\odot A$ upon the ground plane.

Fix any number of points upon the curved line of the near end of cylinder as a, b, c, d, e, f, g, h, and find points upon the ground immediately below them, as b' c', g' h'.

Now imagine vertical planes to pass through $\odot A$ and contain the points on the circumference of the cylinder.

These vertical planes cut the plane of the near end of cylinder in the dotted lines b d, a e, f h, they likewise cut the ground-plane in the chain-lines drawn from A' through the points g', h', a, b', c'.

Draw the rays of light $\odot A$ d", $\odot A$ c", &c., to meet the ground upon the chain-lines and draw the elliptical shadow of the near end of the cylinder through the points a, c", d", e', &c.

Finally, obtain the elliptical shadow of the farther end of the cylinder, and draw a tangent to the two ellipses to D 1.

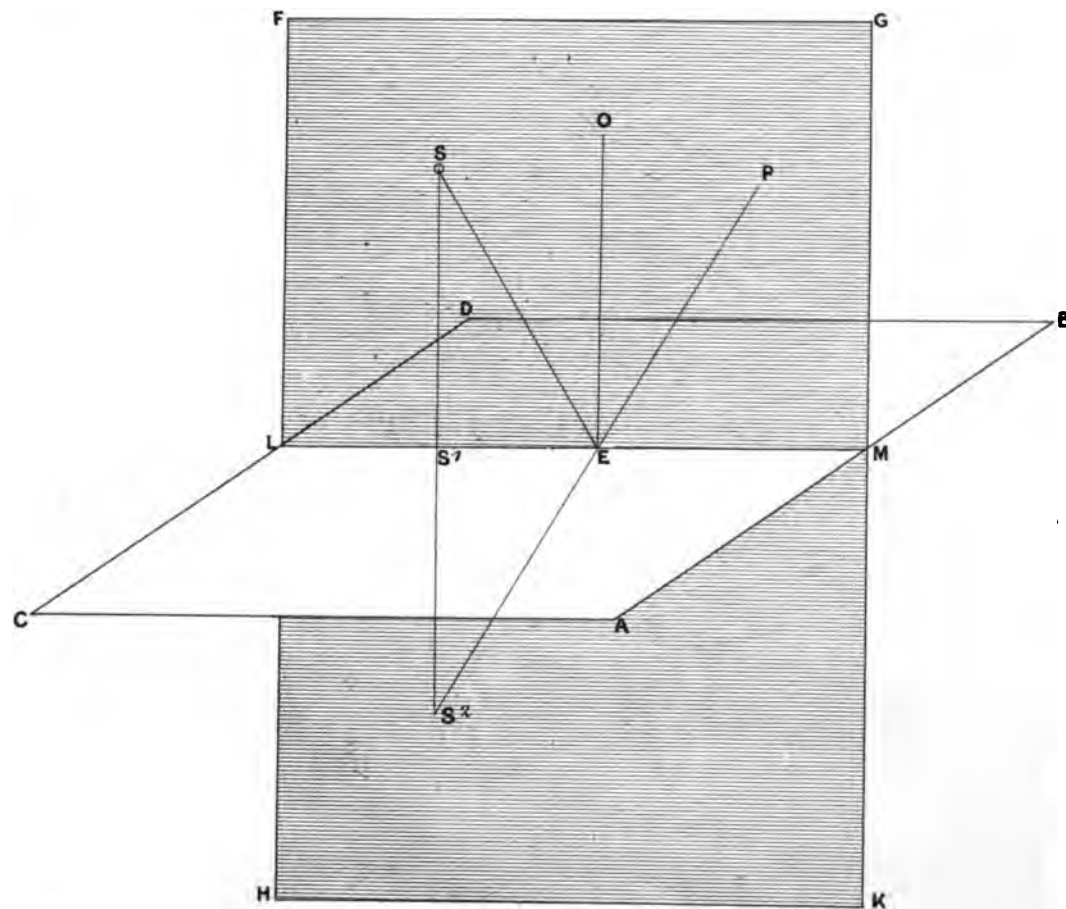


PLATE 60.

REFLECTIONS.

Light is reflected, more or less, from all opaque surfaces to others, but when the surfaces are smooth or polished, the reflections are much more distinctly seen than otherwise.

When the sun's rays come into contact with any reflecting plane they are immediately turned out of their course in the opposite direction to that in which they were impelled at an equal angle with the plane.

Suppose $A B C D$ to represent a horizontal polished surface, and $S E$ a ray of light coming into contact with it at point E .

In order to obtain the reflected image of $S E$ we must imagine a plane perpendicular to the reflecting plane containing $S E$.

$F G H K$ is the representation of the plane containing $S E$, and $L M$ is its intersection with the polished surface.

Let fall a perpendicular from S to meet the polished surface at S' , which is the seat of S .

The reflected image of point E is already obtained, because E lies on the polished surface, but S is some distance above it, therefore its reflected image will appear at the same distance below the plane $A B C D$ as point S is above it.

Produce the perpendicular $S S'$ to S^2 making $S^1 S^2$ equal to $S S^1$.

The line $S^2 E$ is the reflected image of the ray of light $S E$ on the horizontal polished plane.

To prove what has been said above, the student should hold his

pencil in front of a mirror, he will at once perceive that at whatever distance the pencil is from the mirror its reflected image will appear at an equal distance behind it. If the pencil be held at an angle with the surface of the mirror, its reflected image will appear at an equal angle behind it, but inclined in the opposite direction.

The ray of light $S E$ is called an "*incidental ray*." $S^2 E$ is called a "*reflected ray*," and point E is called "*point of incidence*."

$S E O$ is called the "*angle of incidence*," $O E$ is a perpendicular to the polished surface at the point of incidence, and $O E P$ is called the "*angle of reflection*."

N.B. *The angles of incidence and reflection are always equal to one another.*

When an incidental ray meets a polished surface perpendicularly, it and the reflected ray appear one straight line.

By referring to Plates 61, 62, 63, two infallible rules may be deduced.

Rule 1. Lines which are parallel to any polished surface have their reflected images parallel to the original lines; and if the original lines vanish, their reflected images will have the same vanishing point.

Rule 2. Lines which are perpendicular to any polished surface have their reflected images continuations of the original lines, as far beyond the polished surface as the original lines are in front of it.

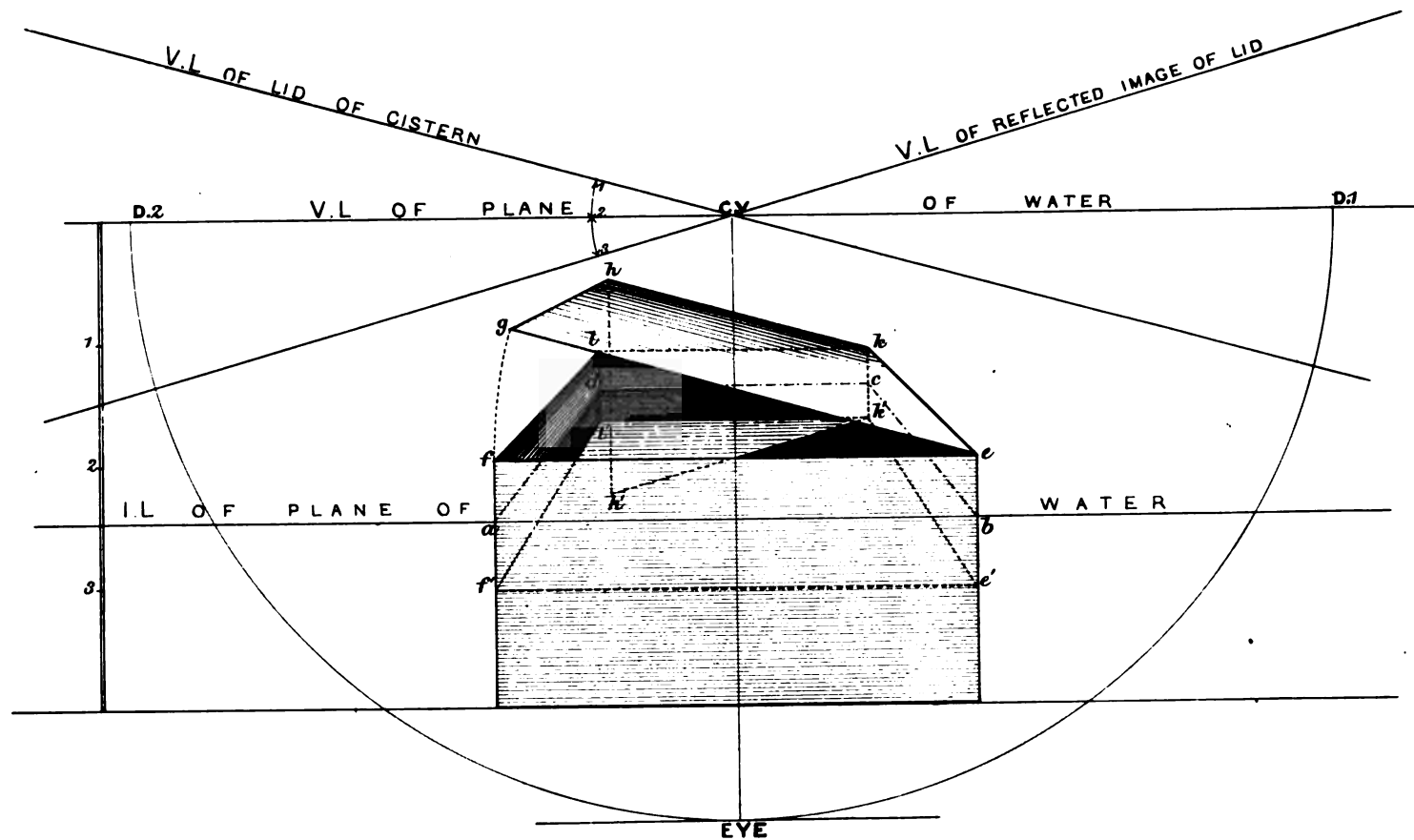


PLATE 61.

Distance of eye in front of the picture-plane, 5'.

Height of eye above the ground-plane, 4'.

Given the perspective representation of a water-cistern 4' square, 2' deep, the lid of which is opened at a certain angle with the ground-plane. The level of the surface of the water is 6" from the upper edges of the cistern.

Required the reflected images of the lid and upper edges of the cistern in the horizontal plane of the water.

The level of the plane of the water is represented by the chain-lines $a d$, $d c$, $c b$.

The upper edge of cistern $f l$ is 6" above the plane of the water and parallel to it, therefore its reflected image must be 6" below the plane of the water and vanish to $C V$. (See Rule 1, Plate 60.) Make $a f$ equal to $a l$. Join f to $C V$.

Then, because $l d$ is perpendicular to the surface of the water, its reflected image must be a continuation of the line $l d$. (See Rule 2.)

Produce the line $l d$ to meet the line $f C V$ at l' .

Make $l' k'$ parallel to $l k$, and determine k' by producing the up-

right edge $k c$. Draw $k' e'$ to $C V$, because the upper edge, $k e$, vanishes to this point.

Having determined the reflected images of the upper edges of the cistern, it will be necessary to find the $V L$ of the plane of the lid.

Draw through $C V$ parallel to the edges $g e$, $h k$, of the lid.

The plane of the lid is found to make an angle with the horizontal plane of the water, *above it*, equal to $1 C V$, 2. We have now to find the $V L$ of reflected image of the lid.

Make an angle *below* the horizon ($2 C V$, 3) equal to that above ($1 C V$, 2).

Then, having found k' , the reflected image of k , we have simply to draw $k' h'$ parallel to the $V L$ of the reflected image of plane of lid.

Point h' is determined by letting fall a perpendicular to the plane of the water from point h .

N.B. The reflected image of h could be found without the use of $V L$ of lid.

Simply ascertain the distance of h above the surface of the water, and make its reflected image (h') at an equal distance below, then join h' to k' .

The former method is far preferable if the student is able to comprehend it.

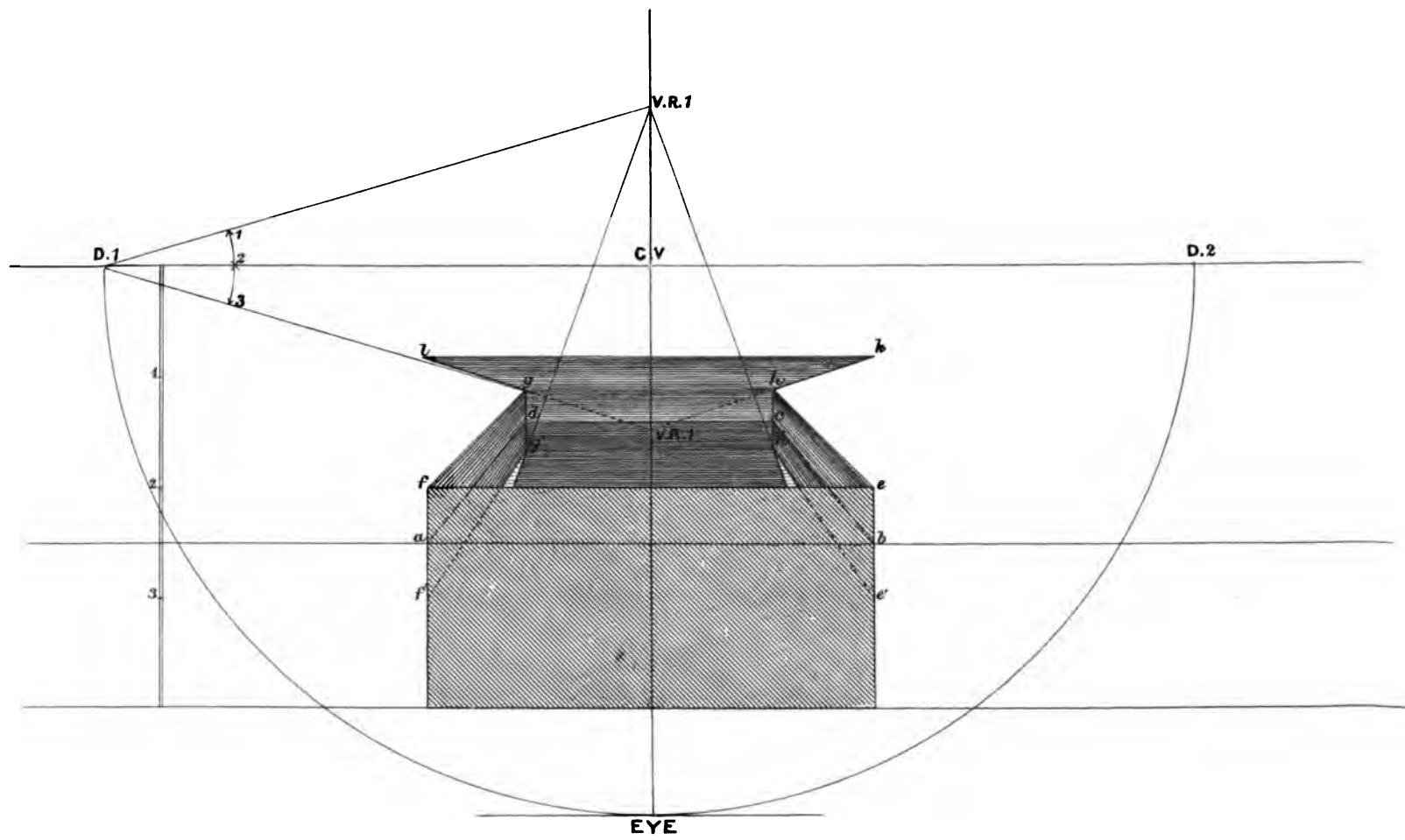


PLATE 62.

Distance of eye in front of the picture-plane, 5'.

Height of eye above the ground-plane, 4'.

Given the perspective representation of a water-cistern (same dimensions as preceding plate), the lid being inclined directly downwards from the spectator at a certain angle with the ground plane.

Required the reflected images of the lid and upper edges of the cistern in the horizontal plane of the water.

The plane of the water is represented by the lines $a d, d e, e b$. The reflected images of the upper edges of the cistern in the surface of the water are obtained by the method given in preceding plate.

N.B. $V E 1$ is found at the same distance above the $V L$ of the plane of the water, as $V P 1$ is below it.

The plane of the lid of the cistern *descends* from the spectator at an angle with the surface of the water equal to $2 D 1, 3$. The edges of the lid $l g, h k$, have $V P 1$ for their vanishing point.

The reflected image of the plane of the lid will appear to *ascend* from the spectator at an angle equal to $1 D 1, 2$ with the $V L$ of the plane of the water.

$V E 1$ is the vanishing point of the reflected images of the inclined edges of the lid.

Join g', h' to $V E, 1$, and produce the lines to meet the near edge, $f e$, of the cistern, because the reflected images cannot be seen beyond this line.

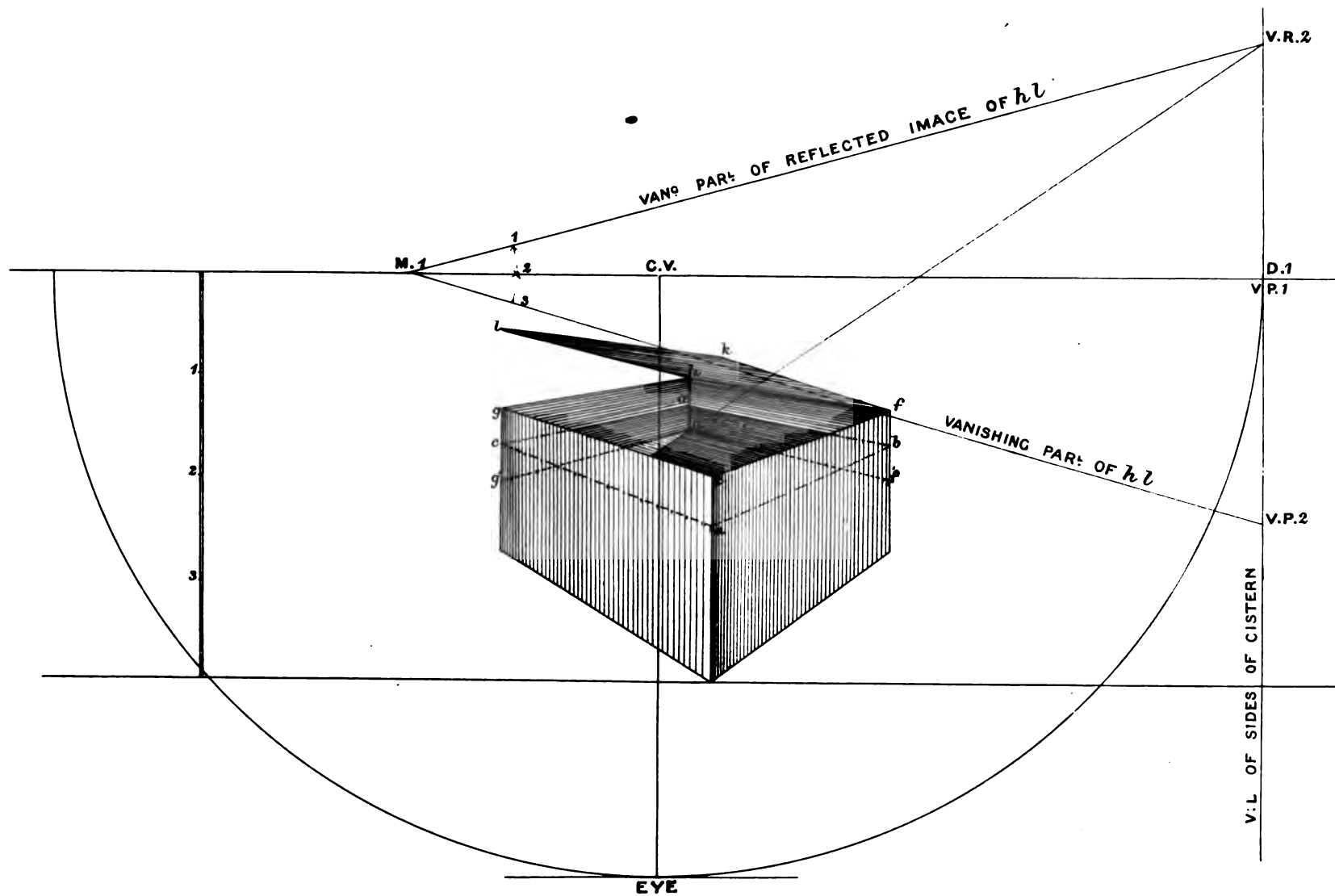


PLATE 63.

Distance of eye in front of the picture-plane, 5'.

Height of eye above the ground-plane, 4'.

Given the perspective representation of a water-cistern (same dimensions as preceding plates) the lid being inclined obliquely downwards from the spectator at a certain angle with the ground plane.

Required the reflected images of the lid and upper edges of the cistern in the horizontal surface of the water.

The surface of the water comes into contact with the vertical sides

of the cistern in the lines $c d$, $d b$, $b a$, $a c$, and the reflected images of the upper edges of the cistern are obtained as previously explained.

Only a portion of the reflected image of the inclined edge of lid ($k l$) is seen by the spectator.

The vanishing parallel of $h l$ descends at the angle $2 \angle 1, 3$ with the $V L$ of the surface of the water, therefore the vanishing parallel of its reflected image must obviously *ascend* at an equal angle, viz. $1 \angle 1 2$.

$V P 2$ is the vanishing point of $h l$, and $V R 2$ is the vanishing point of its reflected image.

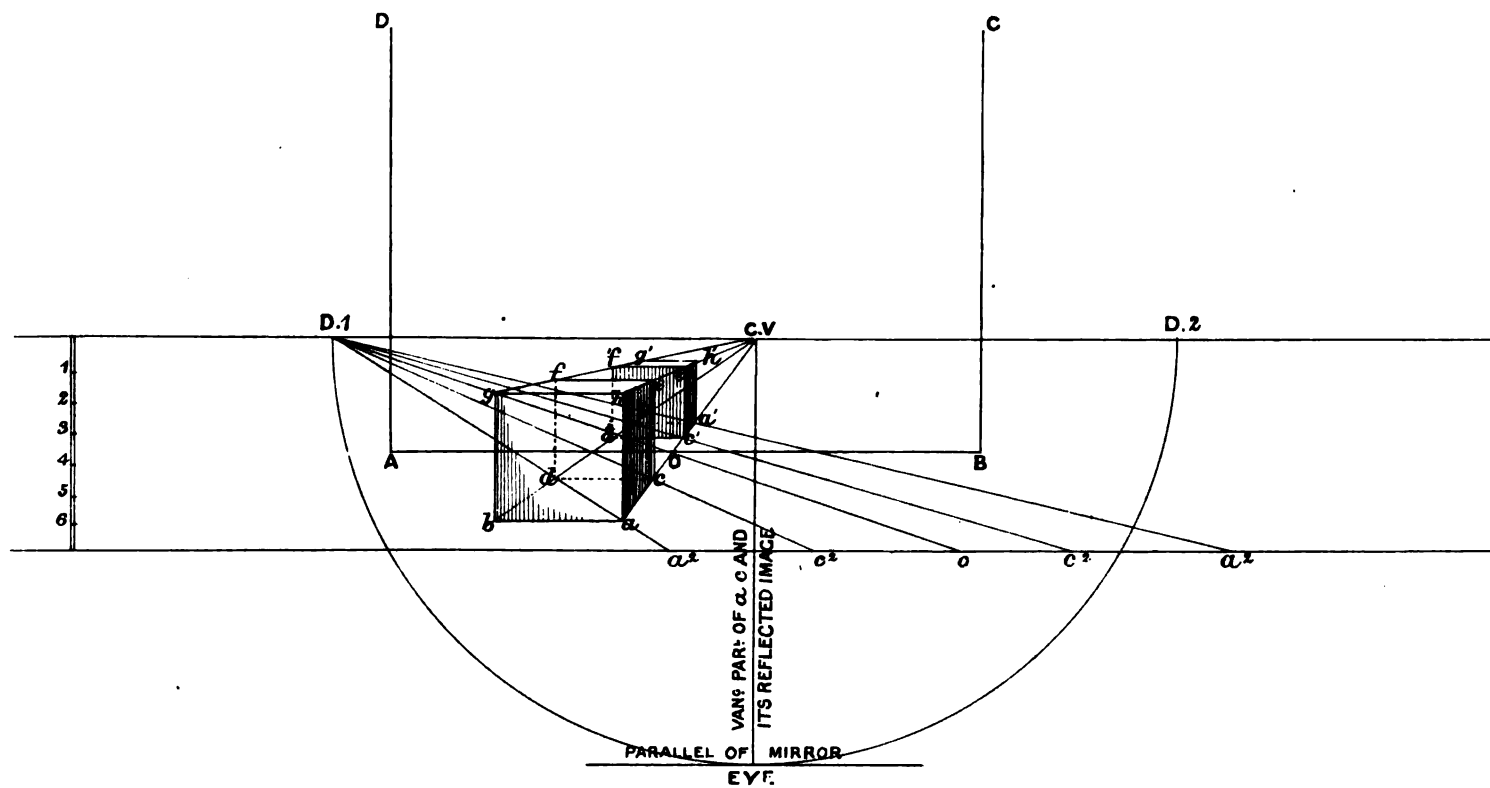


PLATE 64.

Distance of eye in front of the picture-plane, 14'.

Height of eye above the ground-plane, 7'.

Given the perspective representation of a cube of 5' edge having its faces parallel and perpendicular to the picture-plane, and $A B C D$ is the position of a vertical mirror parallel to the picture-plane. Required the reflected image of the cube.

The lower edge of the cube ($e a$) is perpendicular to the plane of the mirror, consequently its reflected image must necessarily be a continuation of the line $e a$.

Find at what distance point e is actually placed in front of the mirror and make the reflected image of that point at an equal distance behind the reflecting plane.

Bring forward O by $D 1$ to meet the ground-line at o , then bring

forward e and a by $D 1$ to meet the ground-line at $a^2 e^2$. Now set off towards right of o (on the ground-line) the distances $o e^2$, $o a^2$, equal to the corresponding distances on the left of o , and join the points to $D 1$, giving $e' a'$, the perspective length of the reflected image of $e a$.

Erect a perpendicular at e' and a' and join e to $C V$, meeting the former lines at e' and h' .

The back face of the cube is parallel to the mirror, therefore the lines forming its reflected image must be parallel to the original lines.

Draw $e' f'$, $e' d'$ parallel to $A B$, then join d and f to $C V$, which will determine the corners d' and f' .

Two of the edges of the upper and lower faces of the reflection of the cube are determined, to complete which we have simply to draw $a' b' h' g'$ parallel to $A B$.

A portion of the reflection of the back face of the cube is hidden by the solid corner h .

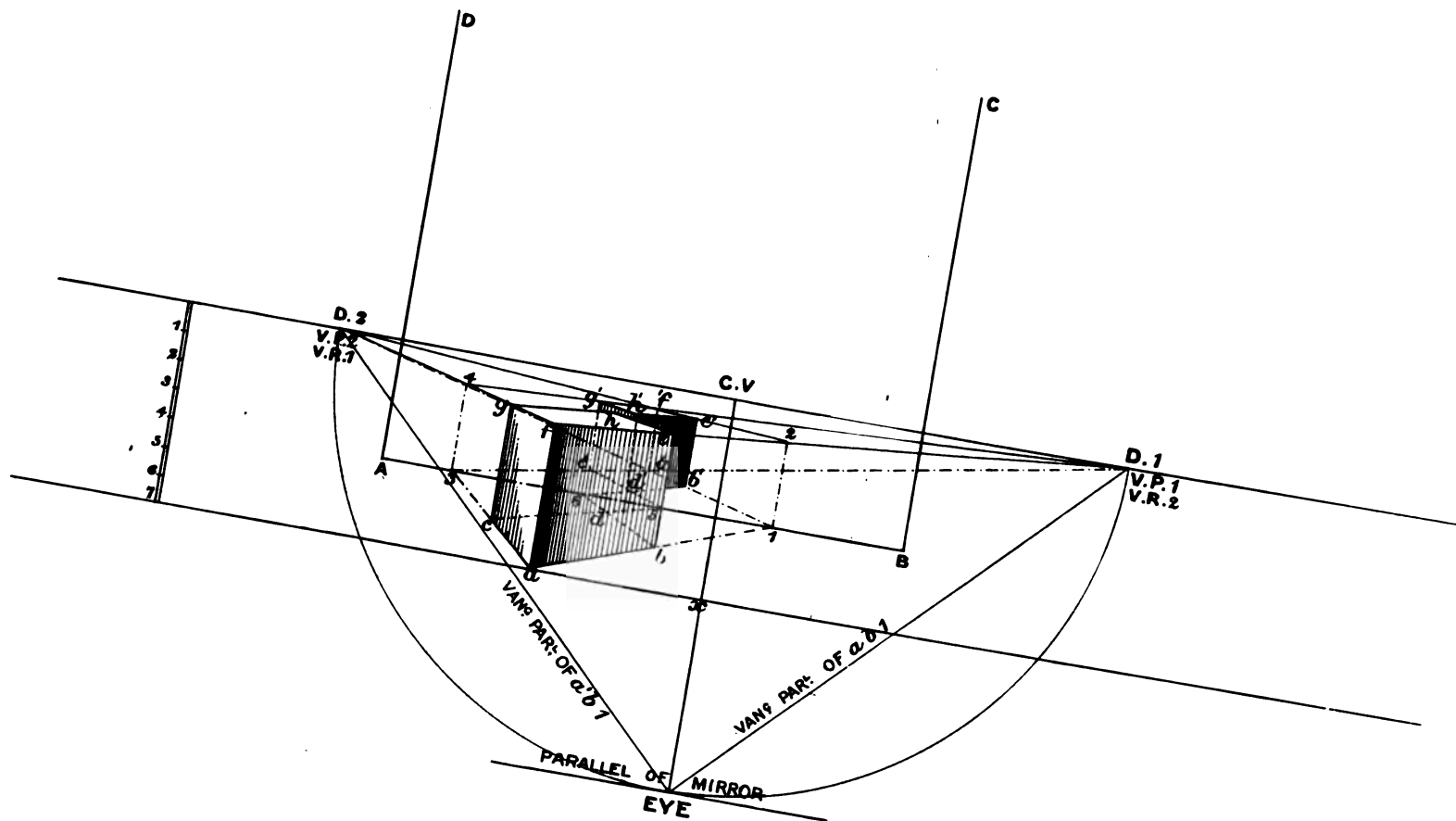


PLATE 65.

Distance of eye in front of the picture-plane, 14'.

Height of eye above the ground-plane, 7'.

Given the perspective representation of a cube of 5' edge, resting on the ground upon its base, the edges of which are inclined to the picture-plane at 45° .

Given the position of a vertical mirror, **A B C D**, parallel to the picture-plane. Required the reflected image of the cube.

The cube has four vertical faces, the planes of which make angles with that of the mirror. It is necessary to produce the vertical planes of the cube and find their intersections with the ground-plane and plane of the mirror.

Produce the vertical face of cube, **a b e f**, to meet the plane of the mirror in the chain-line **1 2**. Its intersection with the ground-plane is the chain-line **a b 1**.

The chain-line **a b 1** is in front of the mirror at an angle of 45° with the picture towards right. The mirror and picture-plane are parallel, therefore if 45° be made with the picture-plane to left, the vanishing point of the reflected image of **a b 1** will be obtained at **V R 1 (D 2)**; join point **1** to **V R 1**, and the reflected images of **a** and **b** must obviously lie somewhere upon it.

Produce **d b** to cut the lower edge of mirror at point **6**. The line **6 d b** has **V P 2** for its vanishing point, it makes an angle of 45°

to left in front of mirror, therefore its reflected image must vanish at an equal angle on the other side.

V R 2 (D 1) is the vanishing point of the reflected image of **6 d b**.

Join **6** to **V R 2**, and where it intersects the chain-line drawn from **1** to **V R 1**, the reflected image **b'** will be determined.

Now produce the edges of the base of the cube (**a c**, **c d**) to meet the lower edge of the mirror at points **3** and **5**.

Because **c a** is actually parallel to **d b** its reflected image has **V R 2** for its vanishing point, and **c d** being parallel to **a b** its reflected image must vanish to **V R 1**, therefore join **3** to **V R 2** and **5** to **V R 1**, the intersections of these lines give the reflected image of the base of the cube.

Produce **f e** (upper edge of cube) to meet the mirror at point **2**. Raise perpendiculars at **a'**, **b'**, and a line joining **2** to **V R 1** will complete the face **a' b' e' f'** of the cube.

Produce the upper edge of the cube, **f g**, to meet the mirror at point **4**, and raise a perpendicular at **c'**. Join **4** to **V R 2**, giving **g' f'** as the reflected image of **g f**.

Finally, draw **g' h'** to **V R 1** and **h' e'** to **V R 2**.

N.B. The reflected images of the corners of the cube may be determined by making their distances behind the mirror perspectively equal those in front of it. The method shown in the plate is preferable.

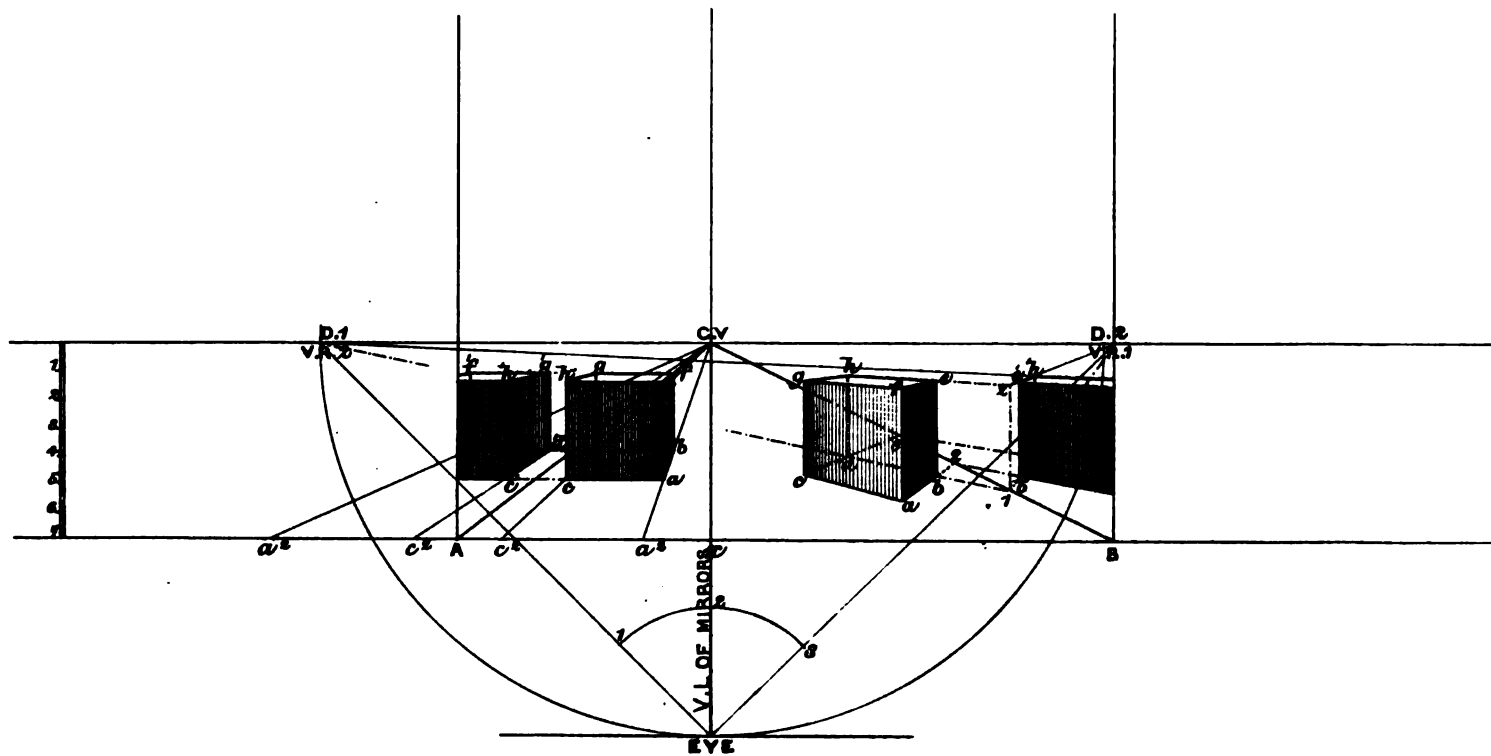


PLATE 66.

Distance of eye in front of the picture-plane, 14'.

Height of eye above the ground-plane, 7'.

I. Given the perspective representation of a cube of 5' edge, having its faces parallel and perpendicular to the picture-plane. A vertical mirror, perpendicular to the picture-plane, is also given. Find the reflected image of the cube.

II. Given the perspective representation of a second cube whose faces are vertical and inclined to the picture-plane at 45° . A vertical mirror perpendicular to the picture-plane is also given to find the reflected image of the cube upon it.

a, b, c, d, e, f, g, h are the given cubes, the chain-lines A C V, B C V, are the intersections of the given mirrors with the ground-plane, and their vanishing line is shown by the vertical line drawn through C V.

TO FIND THE REFLECTED IMAGE OF THE CUBE ON THE LEFT OF THE SPECTATOR.

Bring forward *a b, c d*, by C V, to meet the picture-plane at c^2, a^2 ,

then set off the distances $A c^2, A a^2$, on the left of A equal to the corresponding distances of *a b, c d*, in front of the mirror, and join the points to C V.

Because the lines *a b, c d*, are parallel to the mirror and vanish to C V; their reflected images must also vanish to this point.

Again, because the lines *a c, b d*, are perpendicular to the mirror their reflected images must be in the lines *a c, b d*, produced.

Produce *a c* to meet $c^2 C V$ at the back of mirror at c' ; also, produce *b d* to meet the lines $c^2 C V, a^2 C V$, at the back of the mirror, at points d' and b' .

Raise a perpendicular at points c', b', d' , and produce the upper edges of the cube to meet these perpendiculars at f', g', h' .

TO FIND THE REFLECTED IMAGE OF THE SECOND CUBE.

Produce the vertical face, *b d e h*, of the cube to meet the plane of the mirror in the chain-line 12, its intersection with the ground is the chain-line *d b l*.

The actual angle of *d b l* in front of the mirror at the EYE

is the angle **1 EYE 2**, therefore at the **EYE**, on the other side of the **V L** of mirror, make the angle **2 EYE 3** equal to the former angle, which gives **D 2** as the vanishing point of the reflected image of **d b 1**.

Join point **1** to **V R 1**. Produce the lower edges **a b, c d**, of the cube to meet the lower edge of the mirror at points **2** and **3**. Through points **2** and **3** draw lines to **V R 2**, and where these lines cut **1, V R 1**,

two corners, **b'** and **d'**, of the reflected image of the base of the cube will be obtained.

Erect a perpendicular at **b'** and **d'**, then join point **2** to **V R 1**, giving **e'** and **h'**.

Finally, through **e'** and **h'** draw to **V R 2**, and produce the lines to meet the near vertical edge of the mirror, which completes as much as can be seen of the reflected image of the cube.

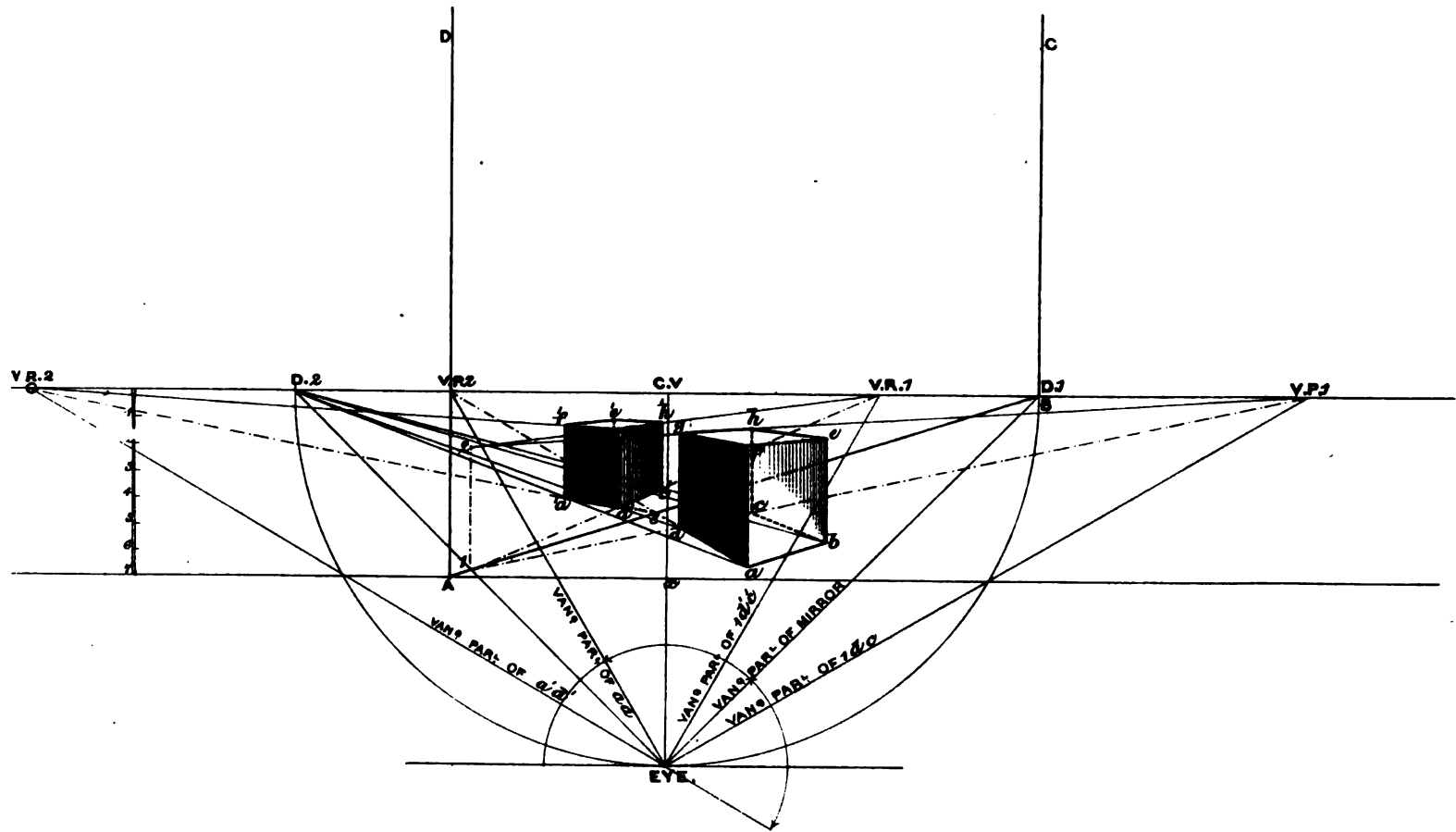


PLATE 67.

Distance of eye in front of the picture-plane, 14'.

Height of eye above the ground-plane, 7'.

I. Given the perspective representation of a cube of 5' edge, resting upon the ground-plane on its base and its vertical faces are inclined to the picture at 30° and 60° .

II. A vertical mirror, $A B C D$, is inclined towards right at an angle of 45° with the picture plane. Required the reflected image of the cube upon the surface of the mirror.

Produce the lower edge of the cube $c d$, by $V P 1$, to meet the intersection of the mirror with the ground plane at point 1.

The line $1 d c$ really makes an angle with the plane of the mirror, equal to that between the vanishing parallels of the mirror and $1 d c$; therefore, if we make the vanishing parallel of the reflected image $1 d' c'$, at the EYE, at an equal angle with the vanishing parallel of the mirror, but on the other side, we shall have the vanishing point of the reflected image of $d c$ at $V R 1$.

Join 1 to $V R 1$, and to determine the images of the corners $d c$ upon this line we have simply to draw from the corners $d c$ perpendicularly to the plane of the mirror.

$D 2$ is the vanishing point of perpendiculars to the plane of the mirror.

Join $d c$ to $D 2$, and the intersections of these lines with $1, V R 1$ will give the reflected images of $d c$.

Now produce the lower edge of the cube $a d$ by $V P 2$, to meet the lower edge of the mirror at point 3.

Find what angle the line $a d$ makes with the plane of the mirror in front, and make an angle equal to it between the vanishing parallels of the mirror and reflected image $a' d'$, but on the opposite side of the mirror.

Produce the vanishing parallel of $a' d'$ to meet the horizon at $V R 2$.

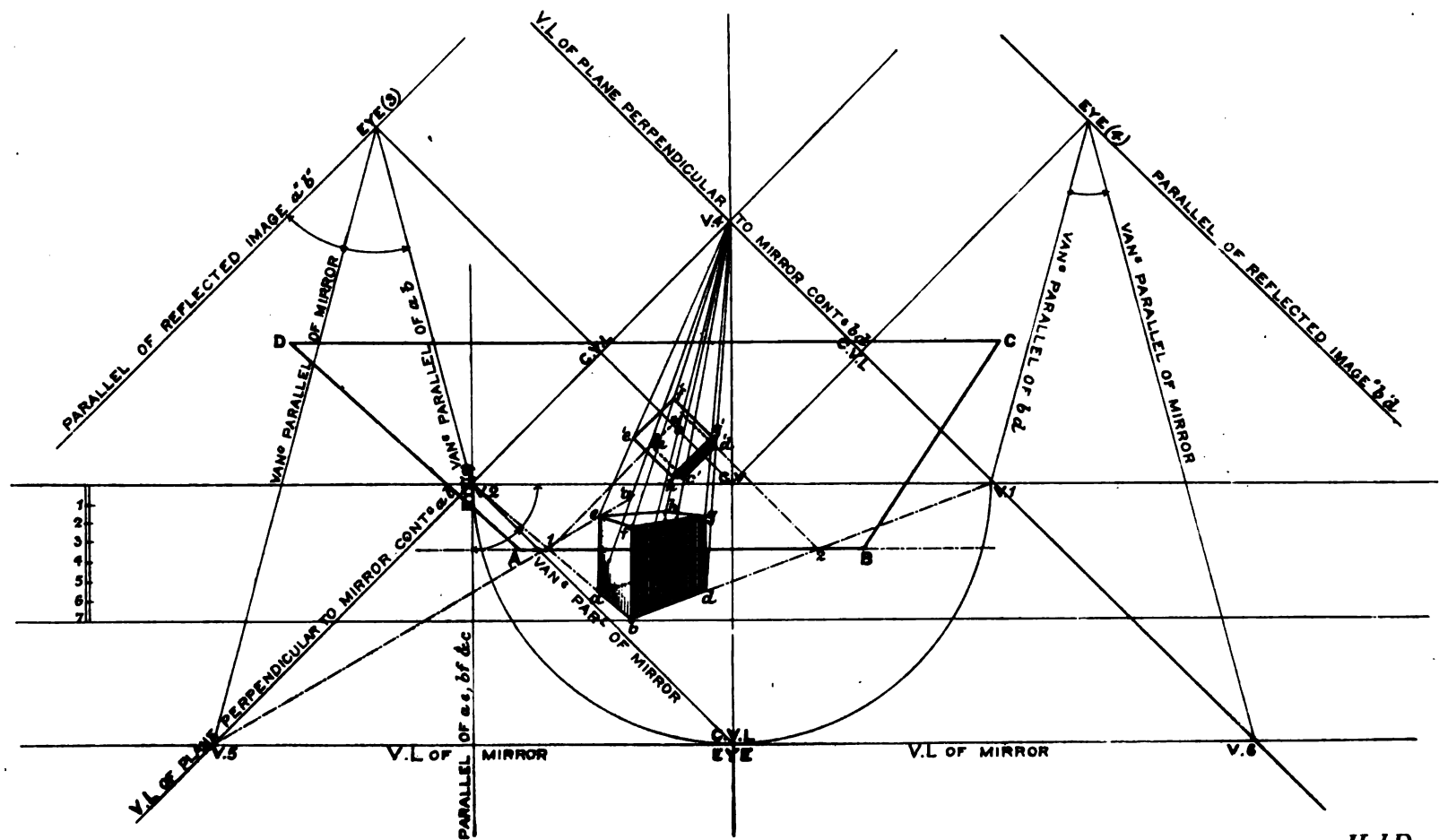
Join point 3 to $V R 2$, then draw a line perpendicular to the plane of the mirror from point a , the reflected image of which is a' .

Draw $a' b'$ to $V R 1$, and $c' b$ to $V R 2$.

Raise perpendiculars at a', b', c', d' , then imagine the vertical face of the cube $d e g h$, produced to cut the mirror in the chain-line 12.

Join point 2 to $V R 1$ to meet the perpendiculars drawn from $d' c'$, at g' and h' .

Draw $g' f', h' e'$ to $V R 2$, and lastly draw $f' e'$ to $V R 1$ to complete the reflected image of the cube.



H.J. Dennis

PLATE 68.

Distance of eye in front of the picture-plane, 14'.

Height of eye above the ground-plane, 7'.

Given the perspective representation of a cube of 5' edge resting upon the ground-plane on its base, having its vertical faces equally inclined to the picture-plane.

ABCD is the perspective representation of a mirror resting upon the ground-plane upon one edge, the plane of the mirror is inclined *directly downwards* at equal angles with the ground and picture-planes. Required the reflected image of the cube upon the plane of the mirror.

It is advisable to find the reflected image of the square forming the base of the cube, then build the cube upon it.

Obtain **V 4**, the vanishing point of perpendiculars to the plane of the mirror, by making a right angle at **EYE (2)** with the vanishing parallel of the mirror.

We must now imagine planes to pass through the lines **a b**, **b d**, perpendicular to the plane of the mirror, and determine their intersections with it.

Join **V 4** to **V 2** and **V 1**, and produce these vanishing lines to meet the **V L** of plane of mirror at **V 5**, **V 6**.

V 5, **V 6** are the vanishing points of the intersections with the mirror of the planes containing **a b**, **b d**, because the vanishing lines of the three planes intersect at these points.

V 1, **V 2** are the vanishing points of the intersections with the ground-plane of the planes containing **a b**, **b d**, because these points are the intersections of the vanishing lines of the planes.

Produce the line **a b** to **V 2**, meeting the lower edge of the mirror at point 1; through point 1 draw a chain-line to **V 5**, and produce it upwards, then draw a perpendicular from **a** to meet the chain-line upon the surface of the mirror at **a'**.

Point **a'** is the "seat" of **a** upon the mirror.

Find the **C V L** of the **V L** of plane containing **a b**, also find **EYE (3)** by the methods given in Plates 18, 19.

We have now to find the actual angle between the chain-lines **1 a**, **1 a'**, and make the reflected angle perspectively equal to it on the other side of the mirror.

Find the vanishing parallel of **a b** by joining **V 2** to **EYE (3)**, then find the vanishing parallel of chain-line **1 a'** by joining **V 5** to **EYE (3)**. The angle between these vanishing parallels at **EYE (3)** is the actual angle of **a 1 a'**.

Measure the angle between the vanishing parallels at **EYE (3)**,

then on the other side of the vanishing parallel of the mirror measure an equal angle which gives the parallel of the reflected image $a' b'$.

It should be observed that the parallel of $a' b'$ is parallel to the $V L$ of the plane containing $a b$; therefore, through point 1 draw a chain-line parallel to the $V L$ just referred to, and upon this chain-line find the reflected images $a' b'$, by drawing perpendicular lines to the mirror from a and b .

Obtain the $C V L$ of the $V L$ of the plane containing $b d$, also determine the position of the eye (EYE 4).

The vanishing parallel of $b d$ is found by joining $V 1$ to EYE (4), and that of the mirror, by joining $V 6$ to EYE (4).

In order to determine the parallel of the reflected image $b' d'$ we have simply to make the angle between it and the parallel of the mirror equal to the angle between the parallels of the mirror and line $b d$.

Since the parallel of reflected image $b' d'$ does not meet the $V L$ of plane containing $b d$, it is obvious that the reflected image cannot vanish, therefore draw from b' a line parallel to $V L$ of plane of $b d$, and it will meet the lower edge of mirror at point 2.

A perpendicular to the mirror from d to meet chain-line $b' 2$ will give d' .

Then, because $a' b'$ is parallel to the $V L$ of the plane containing it, it is manifest that $d' c'$ is also parallel, and for the same reason the line $c' a'$ must be drawn parallel to $b' d'$.

The work may be proved by drawing a perpendicular to the mirror from point c , it will give c' in the position it was previously found.

The upright edges of the cube do not vanish, therefore through EYE (2) draw a vertical line representing their parallel.

The actual angle between the upright edges of the cube and the mirror is shown at EYE (2) between the vanishing parallels of mirror and $a e$, $b f$, &c., and if the parallel of the reflected images of the upright edges of the cube be made at an equal angle but on the other side of the vanishing parallel of mirror, it will coincide with the *horizon*, consequently $C V$ is the vanishing point of these reflected images.

Join a' , b' , c' , d' to $C V$, and to meet these lines join the upper corners of the cube to $V 4$, giving the reflected image of the upper face of the cube at $e' f' g' h'$.

PLATE 69.

Distance of eye in front of the picture-plane, 14.

Height of eye above the ground-plane, 8'.

Given the perspective representation of a square of 5' resting upon the ground-plane on one edge, its plane *descends obliquely* at an angle of 45° with the ground-plane; direction with the picture-plane, 45° towards spectator's right.

The line **A B** is lower edge of mirror inclined to the picture-plane at an angle of 25° to the right of spectator. Required the reflected image of the square when the plane of the mirror *descends* at an angle of 40° with the ground-plane.

Since **V 3** is the vanishing point of direction of the plane of the mirror, its vanishing point of inclination will be found in a vertical plane at right angles to **A B**. At **M 4** make an angle of 40° with the horizon, *downwards*, giving **V 7** the vanishing point of the inclined edges of the mirror.

Find the **V L** of the mirror by joining **V 3** and **V 7**.

Obtain the vanishing point of perpendiculars to the mirror (**V 6**) by making a right angle at **M 4** with the vanishing parallel of the inclination of the mirror.

Imagine a plane perpendicular to the mirror passing through the line **a b**, the **V L** of which is determined by producing a line through the points **V 6**, **V 1**.

Ascertain the positions of **C V L** and **EYE (2)** by the methods given in Plates 17, 18.

The plane perpendicular to the mirror passing through **a b** intersects the ground-plane in the line **a b** produced (chain-line **b o**, **V 1**), it likewise intersects the plane of the mirror in the chain-line **a' o**, **V 8**.

The actual angle between these intersections is found by joining their vanishing points (**V 1**, **V 8**) to **EYE (2)**, and since the reflected image of **a b** is at the same angle behind the mirror as the original line is in front of it, it is obvious that we must make at **EYE (2)** the vanishing parallels of the reflected image **a" b"**, and the original line **a b o**, at equal angles with the parallel of the mirror.

V 9 is the vanishing point of the reflected image **a" b"**.

Join **V 9**, **O** and produce the line, then join the lower corners of the square, **a b**, to **V 6**, giving the perspective length of the reflected image at **a"** and **b"**.

Now imagine a second plane perpendicular to the mirror passing through another side of the square (**a c**).

The **V L** of this plane is found by joining the vanishing point of **a c** (**V 2**) to **V 6**. Find **C V L** and **EYE (3)**.

This plane intersects the ground-plane in the chain-line **a o'**, **V 10**, and passes through the plane of the mirror in the chain-line **o' a' c'**, **V 11**.

Find the vanishing parallels of **o' a'**, **o' a' c'**, by joining **V 10**, **V 11** to **EYE (3)**, then make the vanishing parallel of the reflected image **o' a'** at the same angle with the vanishing parallel of the mirror as the vanishing parallel of **a o'**, but on the opposite side of it.

Through **o'**—draw a chain-line to **V 12**, which will prove that point **a'** was previously found in its correct position.

Find the vanishing parallel of **a c** by joining **V 2** to **EYE (3)**, measure the angle between it and vanishing parallel of mirror, then make a similar angle, but on the opposite side of mirror, for the vanishing parallel of reflected image **a' c'**, and giving **V 13** for its vanishing point.

Join **a'** to **V 13**, and from **c**, the corner of the square, draw a line to **V 6** which gives **c'** as the reflected image of **c**.

The original line **c d** is parallel to **a b**, therefore the reflected image of **c d** must have the same vanishing point as that of **a b**, viz. **V 9**.

The original line **b d** is parallel to **a c**, consequently its reflected image has **V 12** for its vanishing point.

a' b' c' d' is the reflected image of **a b c d**.

N.B. I should have made a cube as in preceding plates, but on account of the *small scale* and the multiplicity of construction lines I have shown a square only.

Every face of a cube is a square, and if the given square be thoroughly understood, the student will be able to complete a cube for himself.

If the reflected image of a curve be required, we have simply to find a number of points in it, and make their reflected images at the same distances on the other side of the mirror as the original points are on this side of it.

Dennis.

PLATE 70.

SOLUTION OF PROBLEM II. ART EXAMINATION PAPER, FEBRUARY, 1879.

Having prepared the paper for the perspective drawing, find **V 1** the vanishing point of a line inclined to picture at an angle of 40° towards left. Upon the ground line set off **5'** on spectator's right and join **5** to **V 1** by a chain line to represent the trace of required ascending plane with the ground-plane. Obtain **V 3**, the vanishing point of inclination of the ascending plane by the method given in *Plate 18*, and draw the **V L** of ascending plane through **V 1**, **V 3**. Find **C. V. L** and **EYE (2)**, join **V 1** to **EYE (2)**. Through point **5** draw the **I L** of ascending plane, and upon this line set off **4'** on left of point **5**, then join **4** to **M 1**, giving the nearest corner (**e**) of the triangular face of the pyramid upon the trace of ascending plane.

Because, the line joining **V 1** to **EYE (2)** is the *vanishing parallel* of the trace of ascending plane, it is obvious that we must make with this line, at **EYE (2)**, an angle of 75° towards right, which determines **V 4**, the vanishing point of the line which forms one side of the triangular face and one side of the base of the pyramid on the ascending plane.

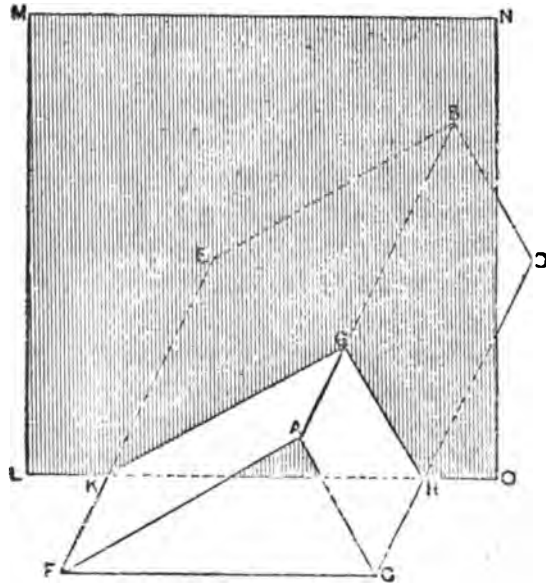
A plan and elevation of the square pyramid must now be drawn so as to ascertain the actual length of the altitude of one of the triangular faces of the solid (see Figs. 1 and 2). Imagine a vertical section of the pyramid made by a plane passing through the centre of the plan (Fig. 1) on the line **c b**. The student should observe that this section plane passes through the *altitudes* **ca**, **a b** of two opposite triangular faces of the pyramid; therefore, if we view the object in the direction indicated by the arrow **d** we shall see the *axis* of the *pyramid* and the *altitudes* of the *faces* of their actual lengths, the former shown by the dotted line and the latter by the lines **a'b'**, **a'c'**, Fig. 2.

Join **e** to **V 4** and measure **e f** perspectively equal to the side of square, Fig. 1. Find point **g**, the perspective centre of **e f**, then draw **g h**, the altitude of the triangle perpendicular to **e f** and perspectively equal to **a'b'**, Fig. 2. **e f g** is the triangular face of the pyramid resting on the ascending plane.

N.B. The inclination of one plane to another can only be determined by a third plane, perpendicular to the other two; or, if a line be drawn in each plane from the same point in their common

intersection, perpendicular to it, the angle made by these lines is the inclination of the planes, for if a plane be made to pass through these lines it will be perpendicular to the common intersection and likewise to both planes.

See accompanying diagram.



K G H (the section of the given planes made by the plane **L M N O**) is the largest angle that can be made by a plane perpendicular to the given planes; consequently, the section **K G H** is the true angle between them.

Let **A B C D**, **A B E F**, be two given planes, their common intersection being **A B**. Let **K G**, **G H**, be the two lines, one in each plane, perpendicular to the common intersection and meeting it at the same point **G**. The plane **L M N O** which passes through these lines **K G**, **G H**, is perpendicular to the given planes, and

TO FIND THE PLANE OF THE BASE OF THE SQUARE PYRAMID.

Imagine a plane perpendicular to the picture-plane to contain the common intersection of the planes of the base and triangular face of the pyramid. The vanishing point of the common intersection is **V 4**, and since the plane to contain it is to be perpendicular to the picture-plane, its vanishing line must necessarily pass through **C. V**; therefore, join **C. V** to **V. 4**. Find **EYE (3)** and draw the vanishing parallel of the common intersection in this plane by joining **EYE (3)** to **V. 4**. With this vanishing parallel, at **EYE (3)**, make a right angle giving **V 6**, the **C. V. L** of the plane which measures the dihedral angle of the planes of the base and face of pyramid. Through **V 6** draw the **V L** of plane last referred to at right angles to **V L** of plane perpendicular to the picture. Find **EYE (4)** and draw the vanishing parallel of triangular face by joining **EYE (4)** to **V 5**. Upon this line construct the elevation of the pyramid resting on its side and having a corner of its base at **EYE (4)**. It is now obvious that if the base of the pyramid be produced, through **EYE (4)**, to meet the **V L** of plane at **V 7**, we shall have its vanishing parallel; and, because **V 4** is a vanishing point of one side of the base and **V 7** another, it is manifest, if these two points be joined, the **V L** of the plane of the base of pyramid will be found. Produce **e f** to meet **I L** of ascending plane, and through the point of intersection draw **I L** of plane of base. Find **C. V. L** and **EYE (5)**, also **M 7** on the **V L** of plane of base.

We have now the **V L** and **I L** of the plane of the base, also the vanishing points of the sides of the base of pyramid; it is, therefore, presumed that the student will not require any further assistance.

EXERCISES.

Distance of eye in front of the picture-plane, 14'.

Height of eye above the ground-plane, 6'.

Scale $\frac{1}{2}$ " to 1'.

I. A rectangle 6' wide, 14' long, lies upon the ground-plane at 4' on left of spectator and 2' beyond the picture-plane, the width of the rectangle is parallel to the plane of the picture. Upon the short side of the rectangle farther from the spectator draw a second rectangle in a *vertical* position 12' high. Join the near corners of the first rectangle to the upper corners of the second, so as to form a triangular prism.

II. A right prism 18' long, $1\frac{1}{2}$ ' square at its ends, rests upon the ground on one of its rectangular faces, the adjacent edges of which make equal angles with the picture-plane; the nearest corner of this solid is 7 on right of spectator, and 9' beyond the picture-plane.

III. A rectangular slab 18' long, 8' high, and 2' thick, stands upon the ground-plane on its thickness, the nearer vertical face of slab is tangential to the back vertical face of square prism, and the thickness of the end of slab is in the same vertical plane as the end of the square prism.

IV. Required the shadow of the first solid upon the other two, also the shadows of the square prism and rectangular slab upon the ground-plane. The sun lies on the left of the spectator in front of the

picture in a vertical plane, making 40° towards right, and the rays of light are inclined to the ground at an angle of 45° . N.B. The length of square prism recedes towards left.

Distance of eye in front of the picture-plane, 16'.

Height of eye above the ground-plane, 5'.

Scale $\frac{1}{2}$ " to 1'.

A semi-cylindrical tube stands upon the ground-plane on its end, the plane of its thickness is vertical and makes an angle of 45° with the picture-plane towards right, and is placed nearest the picture that the spectator may view the interior of the tube. The nearest vertical edge of its thickness is 4' on left of the line of direction and 4' beyond the picture-plane. Thickness of the tube 2', external diameter 12', and height 8'.

II. Upon the upper end of the tube rests a disc (circular plate) 14' diameter, having a circular opening of 4' diameter in its centre. The centre of the disc is placed immediately over the centre of the tube, in which case the curved line of disc projects equally beyond the curved surface of the tube.

III. A rectangular slab 16' long, 7' high, $1\frac{1}{2}$ ' thick, stands upon the ground-plane on one of its long rectangular faces forming the thickness. It is tangential to the curved surface of the tube, and re-

cedes from the picture-plane at an angle of 45° towards right. Its near corner upon the ground-plane is 12' on left of the spectator.

IV. A pole 20' long rests upon the ground plane and leans against the slab, it lies in a vertical plane parallel to the near end of the slab, at 3' from it. Give the perspective representations of the solids and show the shadow of the disc with circular boring upon the thickness and interior surface of the tube and ground-plane, also the shadow of the pole upon the ground-plane and exterior surface of the tube. The sun is to be in front of the picture on left of spectator at an altitude of 30° , and the rays of light are in vertical planes making 55° with the picture-plane.

Distance of eye in front of the picture-plane, 14'.

Height of eye above the ground-plane, 5'.

Scale $\frac{1}{2}"$ to 1'.

I. A right prism 6' long, 2' square at its ends, lies upon the ground-plane on one of its long rectangular faces, the planes of its ends are perpendicular to the picture-plane, that nearest the spectator being 4' on his left and 6' beyond the picture-plane.

II. A right cylinder 4' diameter, 8' long, has its near end at 3' to the left of the eye, and its axis parallel to the picture-plane. The cylinder lies upon the ground-plane on its side, and is in contact with the back vertical face of the square prism.

III. A hexagonal prism 3' side of base, 4' long, rests on the top of square prism and against the cylinder. Its near end is in the same vertical plane as the near end of square prism, and one of the long edges of hexagonal prism is in the centre of the upper face of the first solid.

IV. The ground-plane is supposed to be a polished surface similar to a mirror, required the reflected images of the three solids upon this reflecting plane.

Distance of eye in front of the picture-plane, 14'.

Height of eye above the ground-plane, 7'.

Scale $\frac{1}{4}"$ to 1'.

I. A right prism 6' long, having squares of 2' for its ends, lies upon the ground on one of its rectangular faces, the nearest corner of which is at 2' on right of spectator and 5' beyond the picture-plane, the long edges of the prism recede from the picture-plane at an angle of 40° towards right.

II. An octagonal prism 8' long, 2' edge of base, lies upon the ground on a rectangular face at 3' beyond the square prism. The long edges of the two solids are parallel, and the plane of the near end of square prism is midway between the ends of the octagonal prism.

III. In the centre of the upper surface of the square prism place a regular pentagonal pyramid in such a position that one edge of base is midway from the long and short edges of that surface, and the plane of the base rests upon the octagonal prism. A vertical plane containing the altitude of pentagonal base is parallel to the ends of both the prisms. Edge of base 3', height of pyramid 6'.

IV. Give a perspective representation of the cast shadows of the three solids when the sun is in front of the picture-plane, on the right of spectator, its rays make 30° with the ground-plane, and lie in vertical planes at 35° with the picture-plane.

PERSPECTIVE THEORY.

Art Examination Paper, February, 1877.

I. Define the terms "original line," "angle of vision," "picture plane," "vanishing point," and "traces of a plane."

II. Explain the principle by which measuring points are found, and prove that they really perform the office of measuring given distances on lines going to their respective vanishing points.

III. In what manner would you express the position of a plane inclined at once to the picture and ground planes? Employ definite angles in your illustration giving the number of degrees.

IV. When the perspective representation of a circle is neither a right line nor a circle, what is the actual form of its projection on the picture-plane?

V. Why is the angle of vision generally fixed at 60° ? Mention any reasons you may have for or against enlarging or reducing it.

VI. What is the "horizontal line," and to which plane or planes do you consider it to belong?

VII. Is it theoretically necessary that the picture-plane should be perpendicular to the line of direction? if not, mention any departure from the usual mode of proceeding when otherwise placed and difference in the result obtained.

VIII. Explain the principle for finding the vanishing points of the shadows of right lines when cast on horizontal, vertical, or inclined planes.

Two hours allowed.

Distance of eye in front of the picture-plane, 14'.

Height of eye above the ground-plane, 5'.

Scale $\frac{1}{2}$ to 1'.

I. A pyramid 12' square at base has an altitude of 4'. Place this pyramid in perspective when resting upon the ground on one of its triangular faces. Its base is nearest spectator and lies in an oblique ascending plane, the *direction* of which with the picture is 30° towards right, and its nearest corner is 4' to the spectator's left and 4' beyond the picture-plane.

II. Give perspective representation of a regular hexagonal pyramid, edge of base 4', altitude 8'. The plane of its base coincides with that of the square pyramid, and the lowest angle of its base is coincident with the centre of the edge of first pyramid upon the ground. Two adjacent edges of the base of hexagonal pyramid meet the ground-plane upon the lower edge of base of square pyramid and make equal angles with it.

III. Let the base of the square pyramid and the ground-plane be two reflecting planes, obtain the reflected image of the hexagonal pyramid in these planes.

Distance of eye in front of the picture-plane, 12'.

Height of eye above the ground-plane, 5'.

Scale $\frac{3}{4}$ " to 1'.

I. On the ground-plane, at 2' to left of spectator, and 4' beyond the picture-plane, place the nearest corner of the base of a square slab, one edge of which rests upon the ground and its plane ascends obliquely,

inclination to ground 50° , direction with the picture-plane 50° towards left. Side of square 7'. Thickness of slab 1' 6".

II. In the centre of the upper face of the square slab rests a right cylinder, 5' diameter, 5' high, and to support these solids place underneath the square slab an equilateral triangular prism lying upon the ground-plane on one of its rectangular faces; the near end of this prism lies in the same vertical plane as the near inclined face of the square slab. Altitude of prism 3', length 10'.

III. Required the shadows of these solids when the sun is in the plane of the picture, on the right of the spectator, its rays being inclined to the ground-plane at 30° .

Distance of eye in front of the picture-plane, 12'.

Height of eye above the ground-plane, 5'.

Scale $\frac{3}{8}$ " to 1'.

I. Place in perspective the three solids in the last exercise, in the same positions, and of the same dimensions.

II. Draw in perspective a vertical plane *perpendicular* to the picture-plane, at 8' on the left of the spectator. Let this be a reflecting plane, and show the reflected images of the three solids upon it.

Distance of eye in front of the picture-plane, 14'.

Height of eye above the ground-plane, 6'.

Scale $\frac{1}{2}$ " to 1'.

I. A prism 6' long equilateral triangles of 4' sides for its ends,

passes through the centre of a hexagonal slab whose side is 4' and thickness 2'. The prism projects equally from each face of the slab and its long edges lie in the same planes with three alternate short edges, forming the thickness of the slab. Give a perspective representation of the solids when the prism rests upon the ground-plane on one of the edges of its base, and the slab upon one of its corners. The edge of the prism which rests upon the ground recedes from the picture towards spectator's left at an angle of 45° , and its near extremity is at 4' on left and 4' beyond the picture-plane.

II. Required the cast shadows of the solids upon the ground-plane, the sun to be behind the picture-plane on right of spectator at an altitude of 55° , the rays of light lie in vertical planes making 60° with the picture-plane.

III. The I L of a vertical plane is at 12' on the right of the spectator, this vertical plane recedes from the picture towards left at an angle of 60° . You are required to find the reflected images of the solids upon the plane supposing it to represent a polished surface, as a mirror.

N.B. A second drawing is required for the reflection.

Distance of eye in front of the picture-plane, 14'.

Height of eye above the ground-plane, 8'.

Scale $\frac{1}{2}$ " to 1'.

I. The front elevation of a beer barrel is 6' high, 4' diameter at its ends, and the curved lines forming its sides are arcs of circles of 5' 6" radius. The barrel is composed of 12 staves and bound by two

iron hoops 6" deep, and 6" from each end. Required the perspective representation of the barrel when standing upon the ground plane vertically upon its end, the centre of which is 2' on left of spectator, and 5' 6" beyond the picture plane.

II. Give a second representation of the barrel when lying upon the ground-plane on its side, the plane of its head being *vertical* and inclined to picture plane at 40° towards left. The centre of the head of the barrel (*near end*) is 4' on left of spectator, and 3' 6" beyond the picture plane.

N.B. Separate drawings are required for these problems.

Distance of eye in front of the picture plane, 19'.

Height of eye above the ground plane, 10'.

Scale $\frac{1}{4}$ " to 1'.

I. A semi-cylindrical block of wood, 14' long, 6' diameter, lies upon the ground plane on its rectangular face, the nearest corner of which is 4' on spectator's left and 6' beyond the picture plane, and the planes of its semi-circular ends are perpendicular to the picture plane.

II. Find a point (A) lying upon the ground plane at 4' from the farther semi-circular end of the block of wood, and 3' from the long edge of its rectangular face nearest the spectator.

III. Let point A be the near end of a rod, 14' long, leaning against the solid, and lying in a vertical plane which makes an angle of 60° with the picture plane towards the spectator's left. Distinguish the point of contact of the rod with the convex surface of the solid by the letter B.

IV. Required the shadow of the rod upon the semi-cylindrical block of wood when the sun is in front of the picture plane, on the left of the spectator, in a vertical plane which makes 45° with the picture plane, and the rays of light are inclined to the ground plane at an angle of 30° .

V. A vertical plane parallel to the picture is tangential to the upper extremity of the rod, and a second vertical plane is tangential to the farther semicircular end of the solid. Show the intersections of these planes and imagine them to be mirrors. Give the reflected images of the solid and rod in each reflecting plane.

Distance of eye in front of the picture plane, 12'.

Height of eye above the ground plane, 5'.

Scale $\frac{1}{4}$ " to 1'.

I. A card-basket composed of 6 equal regular pentagons of 5' side (*one half of a dodecahedron*) lies upon the ground plane upon one of its pentagonal sides, the nearest corner of which is 1' to the left, and 3' beyond the picture plane. The base of the card-basket lies in an *oblique ascending* plane, the lower edge of which is upon the ground plane and makes an angle of 50° with the picture plane towards the spectator's left. Give its perspective representation.

II. Required the shadow of the card-basket upon the ground-plane and the interior, when the sun is in front of the picture plane on the right of spectator, its rays making 60° with the picture plane and 30° with the ground plane.

N.B. The V L and I L of each plane of the basket to be obtained.

Distance of eye in front of the picture plane, 16'.

Height of eye above the ground plane, 10'.

Scale $\frac{1}{2}$ " to 1'.

I. An equilateral triangular prism, 6' high, 2' edge at base, stands vertically upon the ground plane, having its near corner immediately below the C V, and 7' beyond the picture plane. One rectangular face of the prism is perpendicular to the picture plane, and the whole of the solid is on the right of the eye.

II. A right prism having square ends of 2', length 8', lies upon the ground plane on one of its rectangular faces, the long edges of which make 60° with the picture plane towards the left, and its nearest corner is 7' on the right of spectator, and 2' beyond the picture plane.

III. A pyramid 6' long, 2' square at base, lies upon the ground plane on one edge of its base, which recedes from the picture at an angle of 60° towards left, the near extremity of that edge is 3' 6" on right of spectator and 3' 6" beyond the picture plane, and the plane of the base of the pyramid descends obliquely from the spectator. One of the inclined faces of the pyramid rests upon the upper long edge of the square prism. Find what angle the base makes with the ground plane, and give a perspective representation of the group of solids.

IV. Required the cast shadows of the solids upon each other and the ground plane, when the sun is behind the picture, on spectator's left, and lying in a vertical plane which makes 28° with the picture plane. The rays of light make 35° with the ground.

Distance of eye in front of the picture plane, 18'.

Height of eye above the ground plane, 6'.

Scale $\frac{1}{2}$ " to 1'.

I. A wall 18' high, 9' thick, touches the picture plane at 6' on right of C V. It is of indefinite length and recedes from the picture plane at an angle of 40° towards left. In this wall you are required to show two semi-circular archways, the centre of the first opening is 7' from the intersection of the wall with the picture plane, and there is a space of 11' between the two centres. The height of each opening is 10' from the ground to the *springing line* of the semi-circular arches, and the width of the archways is 8'.

II. Required the shadow of the curved line of archways upon the interior cylindrical and plane surfaces, supposing the sun to be in front of the picture plane, on the right of spectator and lying in a vertical plane which makes an angle of 45° with the plane of the picture, the rays of light being inclined to the ground plane at an angle of 30°.

III. Imagine the ground to be a horizontal reflecting plane (still water) and find the reflected images of the arches and cast shadows.

Distance of eye in front of the picture plane, 14'.

Height of eye above the ground plane, 7'.

Scale $\frac{1}{2}$ " to 1'.

Give the perspective representation of a wall similarly placed, and of the same dimensions as that given in preceding exercise. In this wall you are required to draw a niche, its base is a semi-circle of 8' diameter, and 3' from the intersection of the wall with the ground,

the nearest corner of the base is 6' from the intersection of the wall with the picture plane. The height of the niche in the centre of the opening in the face of the wall is 14'.

II. Required the shadow of the contour of the niche upon the

interior semi-cylindrical and spherical planes. The sun being in front of the picture-plane, on the right of the spectator, in a vertical plane, making 60°, and the rays of light are inclined at 35° to the ground plane.

ADVANCED ART LOCAL EXAMINATION.

Perspective, May, 1877.

GENERAL INSTRUCTIONS.

If the rules are not attended to the paper will be cancelled.

In all cases the number of the question must be placed before the answer on the worked paper.

The problem is to be worked to a scale of $\frac{1}{4}$ " to 1".

The centre of vision should be in the centre of an imperial sheet of paper.

The distance of the spectator from the picture is 12", and the ground plane 5" below the level of the eye.

Two hours and a half allowed for this exercise.

I. The ground plane is to be divided by a right line starting from the picture line at a point in it 2" on spectator's left and vanishing towards the right at an angle of 40° with the picture line. The portion of the ground plane on the right of the line is to be regarded as still water. At a distance of 6 $\frac{1}{4}$ " from the picture along this line is

the nearest corner of a right prism, the base of which is a right angled isosceles triangle. The prism lies upon the ground plane upon its largest rectangular face, the dimensions of which are 17" \times 12", and one of the 12" edges coincides with the right line dividing the ground plane. Upon the face of the prism which slopes upwards from the water (at 45°) an ordinary brick lies upon one of its largest faces, the edges of which face are inclined at an angle of 15° to those of the face of the prism upon which the brick rests. The dimensions of the brick are 9" \times 4 $\frac{1}{2}$ " \times 2 $\frac{1}{4}$ ". Give the perspective representations.

II. Cast the shadows of these objects when the sun is behind the picture plane on the right of the spectator, and at an altitude of 40°, the rays being in vertical planes inclined to the picture at 30°.

III. Show the complete reflections of the prism and brick with their shadows in the sheet of still water.

ART EXAMINATION PAPER.

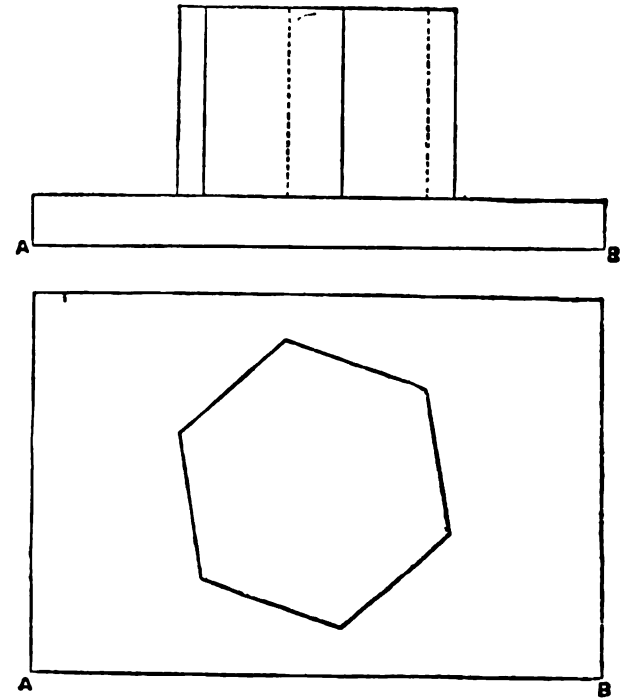
FEBRUARY, 1878.

These problems are to be worked to a scale of half an inch to one foot; the distance of the spectator is to be 12 feet in each instance, and the eye, 5 feet above the ground-plane.

A separate diagram will be required for each problem.

I. A right cylinder of 5' diameter penetrates the ground-plane, and has a length of axis above the ground-plane of $5\frac{1}{4}'$. The axis is inclined to the ground-plane at 55° , and lies in a vertical plane which recedes from the picture-plane at 30° towards the right hand, and meets the ground-plane at a point 2' on the spectator's left and 5' from the picture-line. Give the perspective representation of the visible portion of the cylinder.

II. Give a perspective representation of the slab and prism which are drawn to half the required scale in the accompanying plan and elevation. The edge of the slab **AB** is to lie in the ground-plane, vanishing towards the right hand at 40° with the picture-plane, and the point **A** is 1' on the spectator's left and 1' from the picture-line. The large faces of the slab are to be inclined upwards

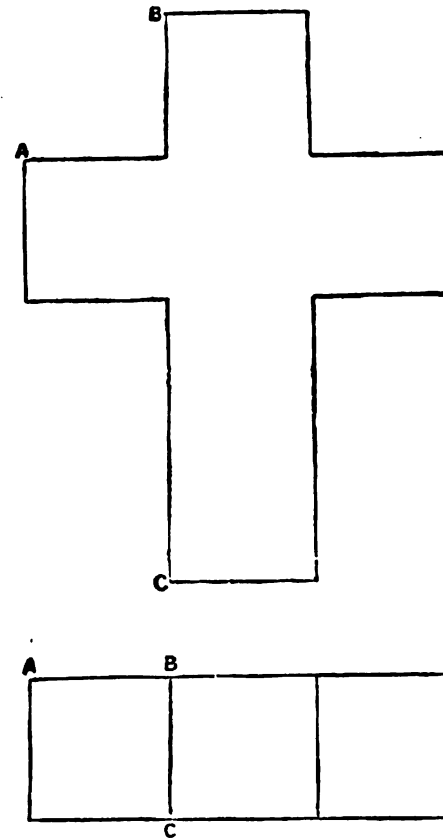


at 30° with the ground-plane, and the two solids are to retain the relative positions shown in the diagram.

(This problem should be worked on a full sheet of imperial paper, upright. The point of sight being 8 inches full scale from the right hand margin and 12 inches from the top).

III. Give a perspective representation of the cross, which is drawn to half the required scale in the accompanying plan and elevation. The cross stands upon the ground-plane, the point C of its base being 4' from the picture-line and 2' on the spectator's left; its faces lie in vertical planes which recede from the picture-plane at angles of 55° towards the right. Cast the shadow of the cross, the sun being in *front* of the picture-plane at an altitude of 30° , and the planes of its rays making angles of 30° with the picture-plane towards the right. Join the points A and B by a thick line and cast its shadow.

Three hours allowed for this paper.



ART EXAMINATION PAPER.

FEBRUARY, 1879.

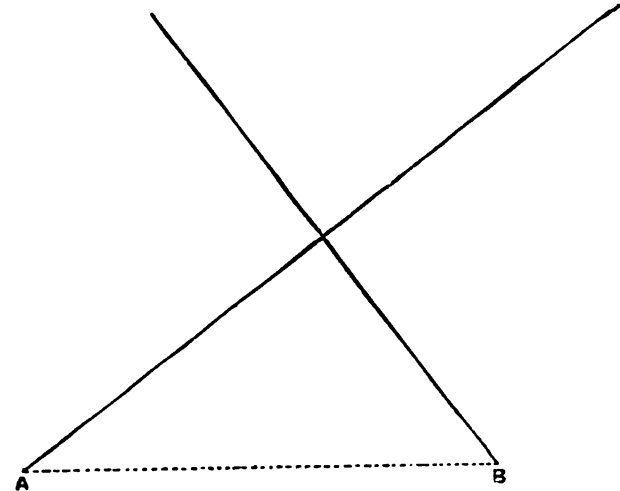
The problems are to be worked to a scale of half an inch to one foot; the distance of the spectator from the picture is to be 12 feet in each instance, and the eye 5 feet above the ground-plane.

A separate diagram will be required for each problem.

I. Two circles of 3' and 4' radii respectively intersect each other at right angles in their horizontal diameters. Their centres coincide. The plane of the larger circle recedes upwards, and that of the smaller, downwards from the picture-plane, and each circle has a point in its circumference resting on the ground-plane. The accompanying diagram is a side elevation of the circles. Point A is to be 4' on the right of the spectator and 5' from the picture-line. The line AB, joining these points of contact, vanishes towards the right at 45° with the picture-plane. Give the perspective representation.

II. An ascending plane rises from the ground-plane at an angle of 30°, and its trace upon that plane cuts the picture-line at a point 5' on the spectator's right hand and vanishes towards the left at an

angle of 40° with the picture-line. A square, right pyramid, the dimensions of which are, height 18' and side of base 6', lies upon



the above plane on one of its triangular faces. The edge of the pyramid, which is one side of this face and also one side of the base, makes an angle of 75° towards the right with the trace on the

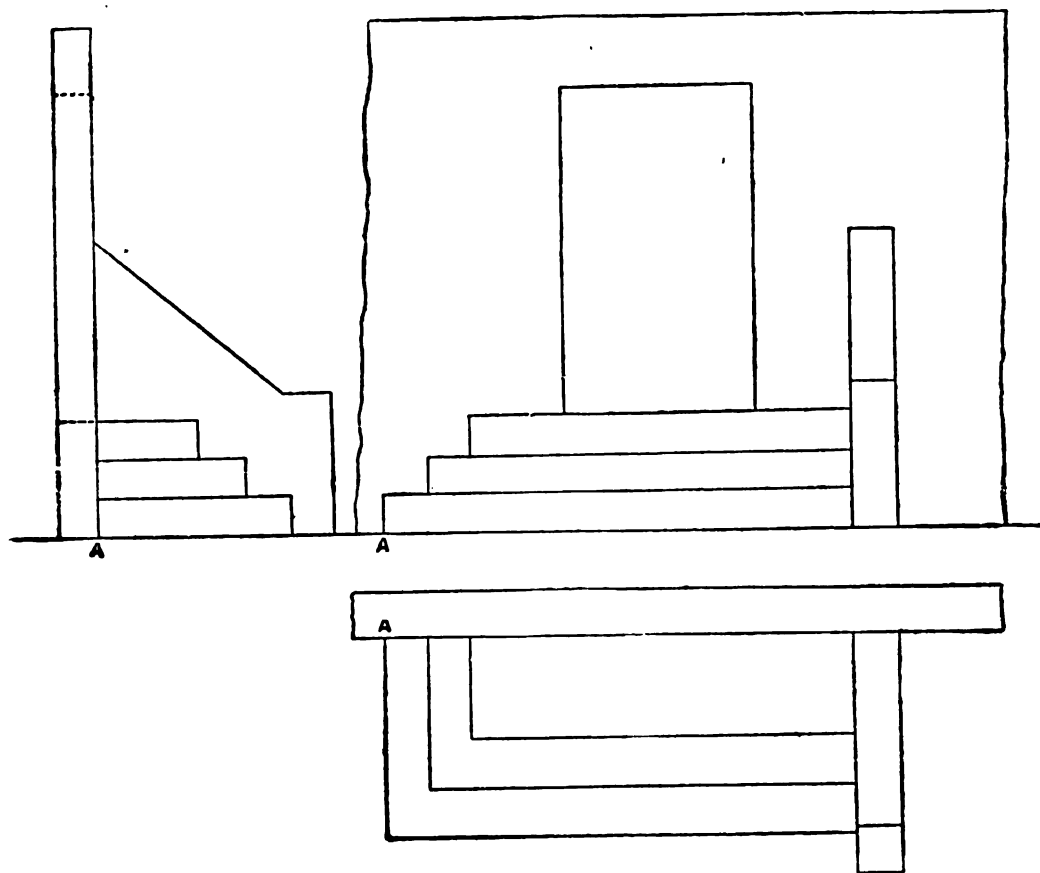
ground-plane of the ascending plane, and one of its extremities lies in that trace at a point 4' from its intersection with the picture-line. Give the perspective representation.

(For this problem, the imperial sheet should be used, upright, and the centre of the picture placed 9 inches from the top of the paper and 12 inches from the right-hand margin.)

III. A short, regular hexagonal, right prism, 3' in length, and having a base of 4' side, is placed with one of its rectangular faces upon the ground-plane. The nearest corner upon the ground-plane is 9' from the picture-line and 2' on the spectator's right hand. The hexagonal faces of the prism lie in vertical planes, making

angles of 40° with the picture-plane towards left. Give the perspective representation of the solid and cast its shadow, the sun being *behind* the picture-plane at an altitude of 40° , and the planes of its rays making angles of 70° with the picture-plane towards the right. At a point upon the ground-plane, 8 on the right and 21' from the picture-line, represent, by a vertical line, a rod 20' in height. Cast the shadow of the rod upon the ground-plane and on the solid, the conditions of the sun remaining as above.

Three hours allowed for this paper.



LOCAL ADVANCED ART EXAMINATIONS.

PERSPECTIVE, MAY, 1878.

GENERAL INSTRUCTIONS.

If the rules are not attended to, the paper will be cancelled.

Put the number of the question before your answer.

The problem is to be worked to a scale of one half-inch to one foot.

The distance of the spectator from the picture is 12', and the ground-plane 5' below the level of the eye.

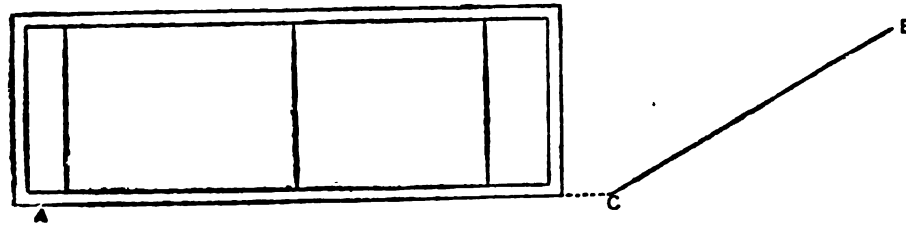
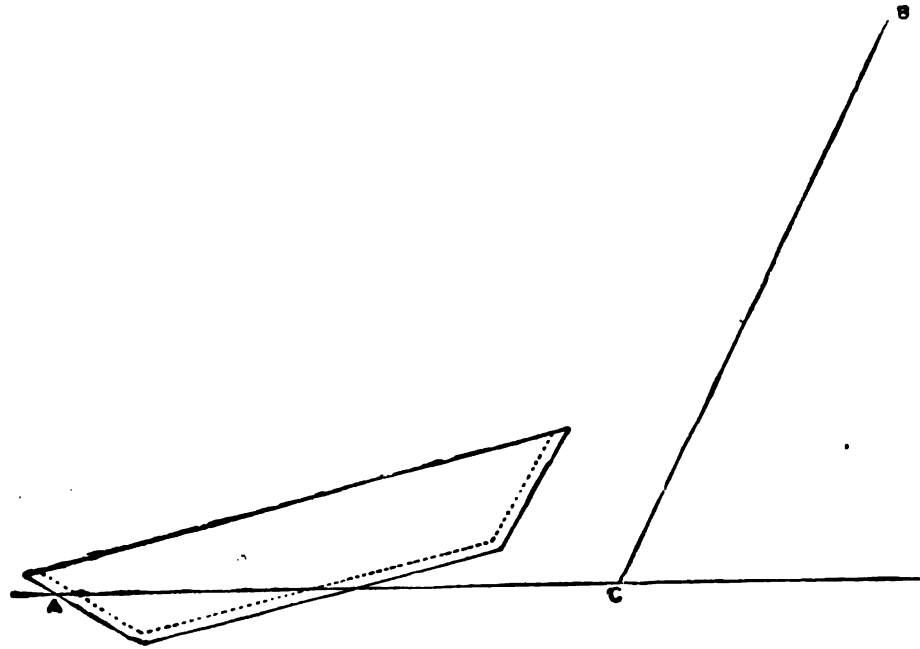
The examination in this subject lasts for two hours and a half.

I. The horizontal trace, on the ground-plane, of a vertical plane in perspective, is drawn from a point on the picture-line 9' on the right of the spectator and recedes from the picture-plane, towards the left, at an angle of 40° with it. A circular plane rests upon the ground-plane, and leans against the vertical plane, the point of contact with the vertical plane being 14' from its intersection with the

picture-plane and 8' above its horizontal trace; the point of contact with the ground-plane being $4\frac{1}{2}'$ from the vertical plane. A square of 4' side is drawn on the plane within the circle and concentric with it. Two sides of the square are inclined at an angle of 25° with the horizontal diameter of the circle, and vanish below the horizon towards left. Give the perspective representation.

II. A vertical garden wall rises from a sheet of calm water, and meets the picture-plane 6' on spectator's left, receding from that plane at 30° towards the right hand. The steps, wall, doorway, &c., drawn to *half-scale*, and shown by plan and two elevations in the accompanying diagram, are to be put in perspective, the point A being 10' along the line formed by the face of the wall cutting the water.

III. Give the reflections and shadows in No. II, the sun being in *front* of the picture-plane at an altitude of 20° , and its rays lying in vertical planes, making angles of 35° with the picture-plane towards the left hand.



LOCAL ADVANCED ART EXAMINATIONS.

PERSPECTIVE, MAY, 1879.

GENERAL INSTRUCTIONS.

If the rules are not attended to, the paper will be cancelled.

Put the number of the question before your answer.

The problem is to be worked to a scale of one half-inch to one foot.

The distance of the spectator from the picture is 12 feet, and the ground-plane 6 feet below the level of the eye.

The examination in this subject lasts for two hours and a half.

I. The ground-plane represents the surface of still water. The accompanying diagram is the plan and elevation, drawn to half the required scale, of a fishing-punt and stake. The punt is partially sunk in the water, which occupies a portion of the in-

terior, the level of the water being the same both within and without. The point A is 2' on the spectator's left hand and 2' from the picture-line, and the sides of the punt lie in vertical planes, which recede from the picture-plane at angles of 60° towards the right. The stake is 14' in length, and vanishes upwards at 60° with the ground-plane. Give the perspective representations.

II. Show the reflections of these objects and cast their shadows, the sun being *behind* the picture-plane at an altitude of 40° , with its rays in vertical planes forming angles of 45° with the picture-plane towards the right.

Candidates must show and mark the vanishing points and vanishing lines of all the lines and planes which represent the punt and stake.

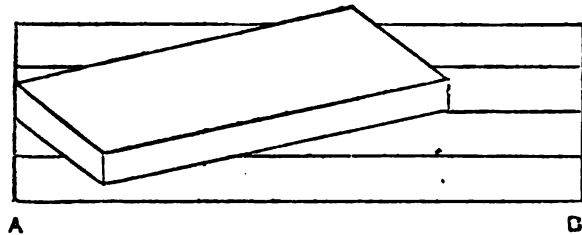
ART EXAMINATION PAPER, GROUP 1.

PERSPECTIVE, FEBRUARY, 1880.

The problems are to be worked to a scale of half an inch to 1 foot; the distance of the spectator from the picture-plane is to be 12 feet in each problem, and the eye 5' above the ground-plane.

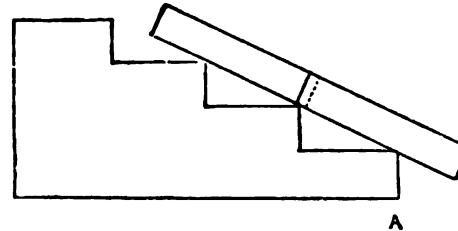
A separate imperial sheet will be required for each problem.

I. A sphere of 6' radius is cut by a plane at a distance of 5' from its centre, and the sphere stands upon the ground-plane, resting on the circular-plane surface created by the removal of the smaller segment. The centre of the sphere is exactly opposite to the centre of vision and 7' from the picture-plane. A second segment is cut off by an oblique plane, which cuts the vertical axis of the sphere at a point 2' above its centre. The horizontal diameter of this sec-



tion vanishes towards left at 50° with the picture-plane, and the diameter at right angles to the horizontal diameter vanishes upwards at an angle of 50° with the ground-plane, and lies in a vertical plane inclined at 40° to the picture-plane towards right. Give the perspective representation of the solid which remains after the two segments have been removed.

II. Two elevations are given of a flight of 4 steps with a rectangular slab laid across them. The diagram is drawn to *half-scale*, and the dimensions of each step are: length 12', breadth 2' and height 1'; the dimensions of the slab are: length 8', width 4', and thickness 9". The point A of the lowest step is to be placed upon the

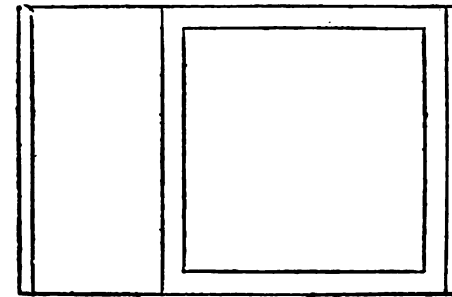


ground-plane, 2' on right of spectator and 3' from the picture-line, and the line AB is to vanish towards right at 30° with the picture-plane, the long edges of the slab are inclined to the long edges of the steps at 25° . Represent the two objects in perspective.

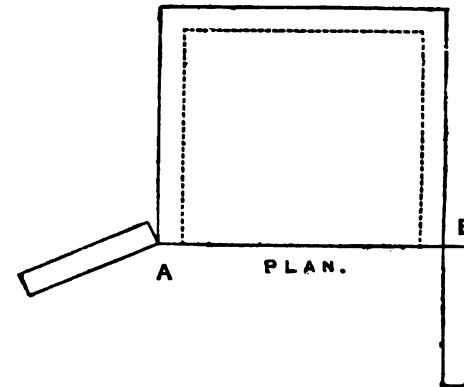
III. The plan and elevation are given to *quarter-scale* of a cupboard with open doors. The point A upon the ground-plane is to be 7' from the picture-line and 4' on spectator's left, and the line AB is to recede from the picture-plane at 30° towards right. Represent the cupboard and doors in perspective, and cast their shadows, the sun being *behind* the *spectator* at an altitude of 30° on the left, and its rays lying in vertical planes vanishing towards the right at an angle of 45° with the picture-plane.

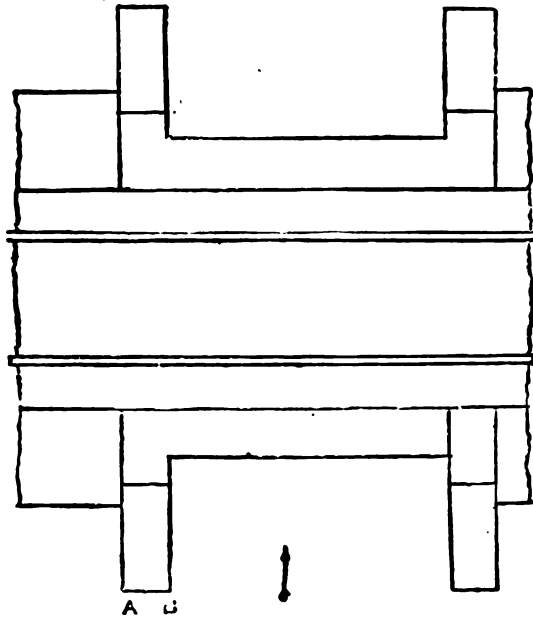
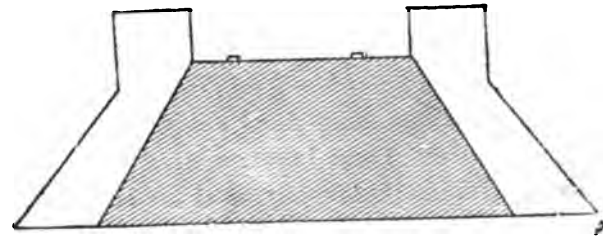
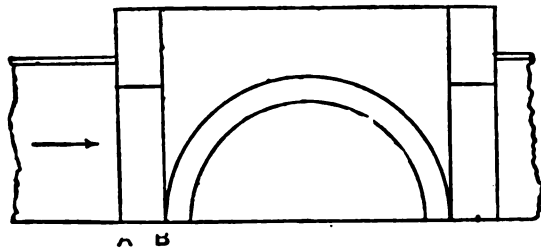
In working this problem an upright imperial sheet should be used, and the centre of the picture be in the centre of the paper.

Three hours allowed for this paper.



ELEVATION.





LOCAL ADVANCED ART EXAMINATIONS.

PERSPECTIVE, MAY, 1880.

The problem is to be worked to a scale of one half-inch to one foot.

The distance of the spectator from the picture is 12 feet, and the ground-plane 5 feet below the level of the eye.

The shadows may either be washed in with a light tint of colour or left in outline merely, but must on no account be shaded with pencil or crayon.

The examination in this subject lasts for two hours and a half.

I. A railway bridge spanning a piece of still water connects two portions of an embankment. The elevation, half sectional elevation taken at right angles, and the plan are given, to *one-fourth*

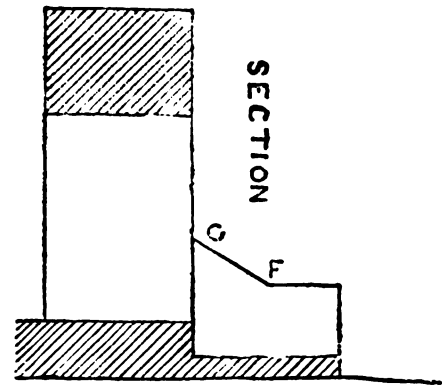
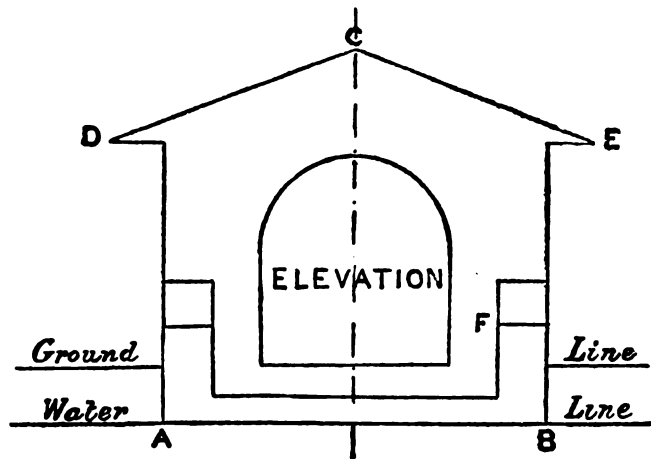
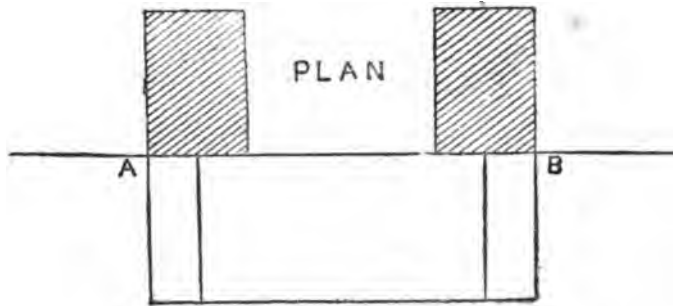
scale. The point A is two feet on the spectator's left, and 3 feet from the picture-line; and the line A B recedes towards the right at an angle of 50° with the picture-line. Give the perspective representation. (*The vanishing and intersection lines of the oblique plane surfaces must be shown and employed.*)

II. Cast the shadows of the subjects in Problem I, the Sun being *behind* the *picture-plane* on the spectator's right, at an angle of 45° , with its rays lying in vertical planes, making angles of 40° with the picture-plane.

III. Suppose the ground on this side of the embankment to be flooded, and give the reflections of both Problems, I. and II. in the water.

LOCAL ADVANCED ART EXAMINATION

PERSPECTIVE, MAY, 1883.



*The problem is to be worked to a scale of two feet to one inch.
The distance of the spectator from the picture-plane is 20 feet, and the "water level" is 7 feet below the eye. The shadows must be indicated IN OUTLINE ONLY.*

The centre of vision should be placed about 3 inches below, and 3 inches to the left of the centre of the paper.

I. The front elevation, cross section, and plan of a water-gate are given to quarter-scale. The perspective representation is required. The point A is to be 2 feet to the left of the spectator, and 12 feet from the picture, and the line AB is to recede from the picture at an angle of 35° to the spectator's right. The lines CD and CE are 10 feet 6 inches long, and are at 20° to the horizontal plane, and the line FG is inclined at 30° to the horizontal plane. The vanishing points of oblique lines should be used.

II. Cast the shadows of the above, the sun being behind the picture on the right at an altitude of 40° , and its rays lying in vertical planes inclined at 60° to the picture-plane.

III. Give the complete reflections of Nos. I. and II.

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